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A Bayesian procedure is presented for estimating the reliability (or availability) of a complex system of independent binomial series or parallel subsystems and components. Repeated identical components or subsystems are also permitted. The method uses either test or prior data (perhaps both or neither) at the system, subsystem, and component levels. Beta prior distributions are assumed throughout. The method is motivated and illustrated by the following problem. It is required to estimate the unavailability on demand of the low-pressure coolant injection system in a certain U.S. commercial nuclear-power boiling-water reactor. Three data sources are used to calculate the posterior distribution of the overall system demand unavailability from which the required estimates are obtained. The sensitivity of the results to the three data sources is examined. A FORTRAN computer program for implementing the procedure is available.

KEY WORDS: Availability analysis; Bayesian statistics; Binomial sampling; Prior distribution; System reliability.

1. INTRODUCTION

Martz, Waller, and Fickas (1988) (henceforth referred to as MWF) presented a Bayesian procedure for estimating the reliability of a series system of independent binomial subsystems and components. They considered either test or prior data (including both or neither) at three or more configuration levels in the system. In this article, this method is extended to arbitrary system configurations of series/parallel subsystems of other subsystems or components. Because of the similarity between this and the earlier MWF procedure, readers are referred to MWF for supporting details. In particular, the procedure presented here uses several MWF definitions and techniques—the notion of native and induced prior distributions (MWF, sec. 2), a beta approximation method for obtaining the induced prior distributions (MWF, sec. 2.2), Monte Carlo simulation for examining the validity of the beta approximation (MWF, sec. 2.3), and a procedure for combining native and induced subsystem prior distributions (MWF, sec. 2.4). Barlow (1985), Cole (1975), Dostal and Iannuzzelli (1977), Mastran (1976), Mastran and Singpurwalla (1978), and Natvig and Eide (1987) considered the problem of obtaining Bayesian estimates of the reliability of either simple or complex series/parallel arrangements of independent pass/fail components. MWF also contains additional references.

The Bayesian procedure presented was motivated by the following problem concerning the engineered safety features of a certain 1,150 megawatts electric U.S. commercial nuclear-power boiling-water reactor. In such a reactor, one important safety system is the low-pressure coolant injection (LPCI) system that provides coolant to the reactor vessel during accidents in which vessel pressure is low. It consists of two trains containing pumps, valves, heat exchangers, and piping. It normally operates in a standby mode, awaiting a demand for its use. Consequently, certain components must perform a change of state on demand; for example, the motor-driven pumps must start, the motor-operated valves must operate, and the check valves must open. Once started, the system must operate for a designated length of time, and, consequently, various time-related failure modes are also of interest. We restrict our attention here, however, to failure to start on demand (or simply failure on demand). In this case the binomial distribution is the appropriate model for the test data and the probability of the system failure on demand is known as its *demand unavailability*. The system may be unavailable on demand for two reasons, failure while on standby and unscheduled maintenance. For simplicity only, however, that portion of demand unavailability caused by standby failure is considered here.

Figure 1 shows the system-demand-availability block diagram corresponding to an accident in which a single pump train would be sufficient to mitigate the accident. The system-demand-availability block diagram thus consists of two trains in parallel, each consisting of two parallel pump trains, both of which operate in series with a motor-operated valve and a check valve. The problem is to estimate the system demand unavailability due to failure while on standby based on tests and prior data on each component, as well as additional prior data on pump trains A–D and LPCI subsystems A–B. Figure 1 also shows the decomposition of the system into a set of 11 series or parallel subsystems required for the model presented in Section 2. This problem is again considered in Section 3.

The situation considered here is one in which a system of conditionally independent components may be decomposed into a set of *m* series or parallel subsystems of other subsystems or components. Some of the components or subsystems may appear more than once in the system. These multiple apperances do not mean that the same device performs more than one function in the system. Rather, the replicated devices represent those that are either known or believed to have identical reliabilities (or availabilities). Such repeated devices often will have binomial test results only about the common generic device and not about each individual physical device in the system. The method accommodates such replicated devices.

A proposed solution to the extended problem is

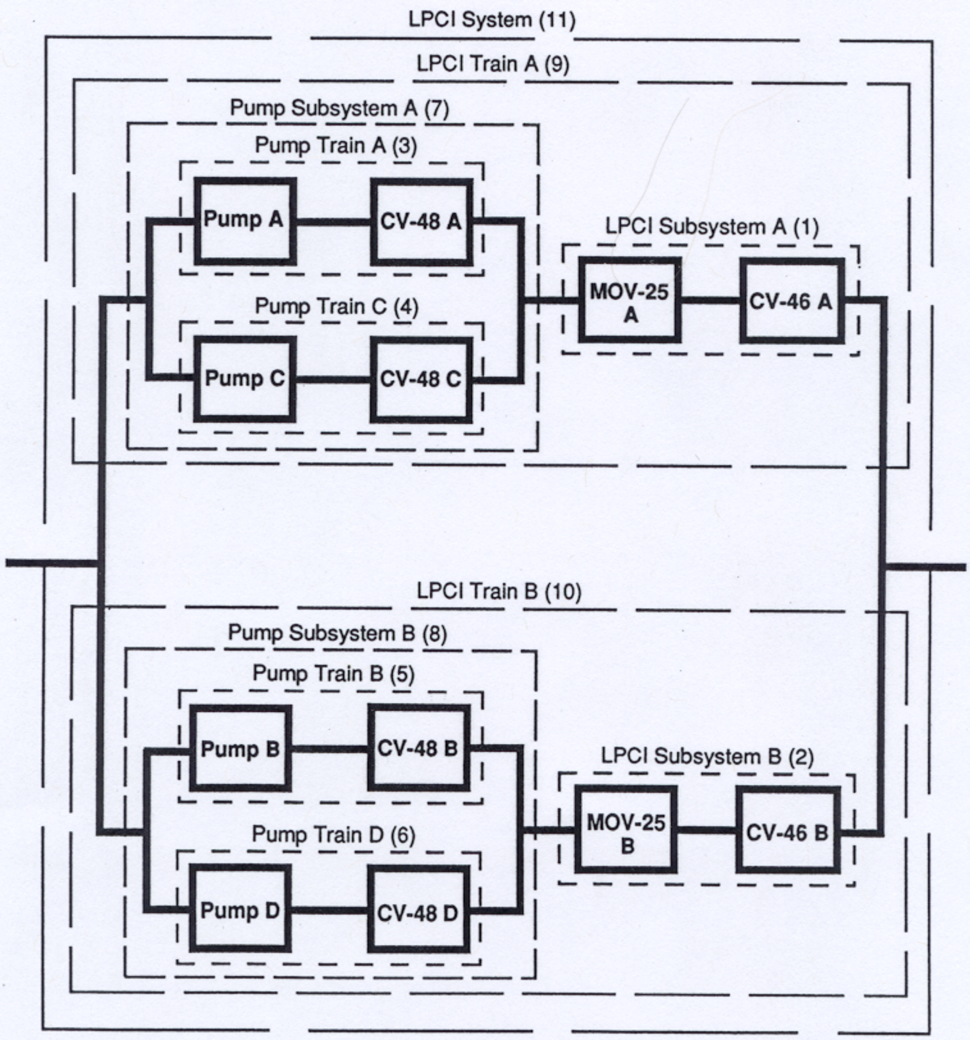

Figure 1. LPCI System-Demand-Availability Block Diagram: LPCI, Low Pressure Coolant Injection; MOV, Motor-Operated Valve, CV, Cheek Valve. The numbers in parentheses are subsystem identifiers.

presented in Section 2. In Section 3 the coolant-injection-system problem is again considered and used to illustrate the method. Some conclusions are presented in Section 4.

2. THE EXTENDED-BAYESIAN-ANALYSIS MODEL

The number of survivors s of a device in n independent tests is assumed to follow a binomial distribution $f(s \mid r; n)$ $(s = 0, \ldots, n, 0 \leq r \leq 1)$, where r is the common survival probability (reliability) of the device across the n tests. The prior distribution for r is taken to be the $\beta(p, q)$ distribution defined by MWF.

The notation and data structure is the same as defined by MWF with one important difference. The label *component* here denotes any subsystem that acts like a component in the procedure. Recall that a subsystem may consist of a series or parallel arrangement of components or other subsystems. Such nested subsystems (when they exist) play the role of components, thus the use of the generalized notion and corresponding label *component*. The system is assumed to be decomposable into a set of m subsystems in which the ith subsystem contains k_i nonidentical (unique) components in series or parallel.

The procedural steps of a Bayesian model for solving the extended problem considered here are quite similar to those of MWF. They consist of (a) decomposing the system into a set of m series or parallel subsystems of other subsystems or components, (b) following steps 1–5 (Stage 1—Subsystem Level Analysis) of MWF, and (c) repeating (b) for each of the m subsystems identified in (a).

The Bayesian procedure as presented can be applied to complex series/parallel systems. The system decomposition ensures that a system under consideration has the requisite series/parallel structure and identifies the procedural order to follow in the analysis. We prefer to perform the decomposition using a reliability block diagram of the system. Because of the complementary relationship between unreliability and reliability (e.g., series reliability logic is equivalent to parallel unreliability logic), unreliability block diagrams can also be used, however.

The decomposition follows a bottom-up approach. It begins with the identification of those subsystems at the lowest level in the system, where *low* means a simple series or parallel arrangement of actual components as opposed to other subsystems. This process of identification continues upward as previously identified subsystems become components in yet higher-order subsystems. The decomposition finally culminates in the identification of the overall system (the highest-order subsystem in the decomposition) as some series or parallel configuration of high-order subsystems (when treated as single entities). This decomposition process provides a structured procedure for the analysis, is easy to do, and, even for complex series/parallel systems, is quite straightforward in practice. It is illustrated in Section 3.

Following a few simple guidelines when performing the decomposition is beneficial later in the Bayesian procedure. The subsystems should all be consecutively numbered 1 to m, with the mth subsystem denoting the system itself (see Fig. 1). Henceforth, when we refer to the mth subsystem, we are referring to the complete system. Moreover, we have found it helpful to construct a table indicating which components are contained in each subsystem by both name and subsystem number. Both input test and prior data, as well as output posterior beta distribution parameters, can also be displayed for each subsystem as they are determined in the course of the analysis.

Further, for simple (pure) series, parallel, series-parallel, or parallel-series systems, there is only a single level of subsystem nesting—the mth subsystem is a pure series or parallel configuration of the lowest-order subsystems themselves. On the other hand, complex series/parallel systems can have many levels of subsystem nesting depending on the complexity of the system.

The following discussion refers to steps 1–5 of MWF, the important differences being the necessary notation to accommodate identical components and parallel structures.

Suppose now that there exist data only about the generic component in each set of identical components and further that each set of identical components always appears together in the same subsystem. This situation occurs frequently when redundant (identical) components are arranged in an active parallel configuration. Let k_i $(i = 1, \ldots, m)$ denote the number of nonidentical (uniquely different) components in subsystem i. Further, let k_{ij} $(j = 1, \ldots, k_i)$ denote the number of replications of the jth nonidentical component in subsystem i. Note that the total number of components in the ith subsystem is $\sum_j k_{ij}$. For example, for the ith simple series subsystem of components given by 1-1-1-2-2-3-4, in which components with the same number are identical, we have $k_i = 4$, $k_{i1} = 3$, $k_{i2} = 2$, $k_{i3} = k_{i4} = 1$, and $\sum_j k_{ij} = 7$. In the LPCI-system example, in spite of the redundancy, each component is assumed to have its own unique underlying availability value and thus there are no identical components ($k_{ij} = 1$ for all i and j).

Two cases are considered. First, suppose that the jth nonidentical component in the ith subsystem is an actual component (as opposed to a lower-order subsystem). For a binomial sampling model $f(s_{ij} \mid r_{ij}; n_{ij})$ and a $\beta(s_{ij}^0 + 1, n_{ij}^0 - s_{ij}^0 + 1)$ prior on r_{ij}, the corresponding posterior distribution is a $\beta(s_{ij} + s_{ij}^0 + 1, n_{ij} + n_{ij}^0 - s_{ij} - s_{ij}^0 + 1)$ distribution. Of course, if there are no binomial test data for the component, then the posterior is the same as the prior distribution. In the absence of prior data, some type of noninformative prior must be used, such as a $\beta(1, 1)$ or $\beta(.5, .5)$ distribution.

Now suppose that the jth nonidentical component in the ith subsystem is a lower-order subsystem that has been previously considered. The bottom-up decomposition of the system ensures that this will be so. The previous analysis on this lower-order subsystem (the component in the current subsystem) produced a subsystem posterior beta distribution for this component. This previously obtained subsystem posterior beta distribution now becomes the prior distribution for the jth component in the ith (current) subsystem analysis. Any subsystem-level binomial test results on this component have already been considered in obtaining the subsystem posterior beta distribution. Therefore, there are no binomial test data for this component in the ith (current) subsystem analysis, and thus $s_{ij} = n_{ij} = 0$. The posterior beta distribution from the previous analysis of this component must of course be expressed in the same component beta prior form as in the case of an actual (true) component. This form will be presented later. Thus the posterior distribution for the jth component in the analysis of the ith (current) subsystem is simply the posterior distribution obtained for this component from the previous analysis. This iterative process will be illustrated in Section 3.

The induced prior distribution for the reliability r_i of the ith subsystem, $g(r_i)$, depends on whether the subsystem is a series or a parallel configuration and the structure of the identical components within the subsystem. All nonidentical components are further assumed to have underlying prior reliability values that are independently distributed. For a series configuration, the ith subsystem reliability expression is given by

$$r_i = \prod_{j=1}^{k_i} r_{ij}^{k_{ij}}. \tag{1}$$

In this case, the induced prior distribution on r_i given in (1) is the distribution of the product of powers of independent beta random variables in which r_{ij} has the posterior component beta distribution given previously. In the case of all nonidentical components, $k_{ij} = 1$ for all j, k_i is the total number of components in the subsystem, and (1) reduces to the usual product-reliability rule. Note that for notational ease we have suppressed the conditional dependence of $g(r_i)$ on the component prior and test data, a convention that we will continue to follow.

Now consider a parallel subsystem configuration. The ith subsystem reliability expression becomes

$$r_i = 1 - \prod_{j=1}^{k_i} (1 - r_{ij})^{k_{ij}}. \tag{2}$$

The posterior distribution of component unreliability $1 - r_{ij}$ is $\beta(n_{ij} + n_{ij}^0 - s_{ij} - s_{ij}^0 + 1, s_{ij} + s_{ij}^0 + 1)$, in which $n_{ij} = s_{ij} = 0$ if the component is in fact a previously analyzed subsystem. Of course, for a true (actual) component, binomial test results may be either unavailable or nonexistent, in which case $n_{ij} = s_{ij} = 0$. The induced prior distribution on r_i in (2) is thus the complementary distribution of the product of powers of independent beta random variables given previously.

Although it is sometimes possible to determine the exact distribution of a product of powers of independent beta random variables, various authors have suggested approximating the exact distribution with a beta distribution having the same first two moments. This approximation was also used by MWF. Using this approximation, the approximate induced prior distribution on r_i, denoted by $g_a(r_i)$, is a $\beta(a_i, b_i)$ distribution in which

$$a_i = M_i(M_i - W_i)/(W_i - M_i^2)$$
$$b_i = (1 - M_i)(M_i - W_i)/(W_i - M_i^2) \tag{3}$$

and where

$$M_i = \prod_{j=1}^{k_i}\left[\prod_{q=1}^{k_{ij}}\left(\frac{s_{ij} + s_{ij}^0 + q}{n_{ij} + n_{ij}^0 + q + 1}\right)\right] \quad \text{for a series subsystem}$$
$$= 1 - \prod_{j=1}^{k_i}\left[\prod_{q=1}^{k_{ij}}\left(\frac{n_{ij} + n_{ij}^0 - s_{ij} - s_{ij}^0 + q}{n_{ij} + n_{ij}^0 + q + 1}\right)\right], \quad \text{for a parallel subsystem} \tag{4}$$

and

$$W_i = \prod_{j=1}^{k_i}\left[\prod_{q=1}^{2k_{ij}}\left(\frac{s_{ij} + s_{ij}^0 + q}{n_{ij} + n_{ij}^0 + q + 1}\right)\right], \quad \text{for a series subsystem}$$
$$= \prod_{j=1}^{k_i}\left[\prod_{q=1}^{2k_{ij}}\left(\frac{n_{ij} + n_{ij}^0 - s_{ij} - s_{ij}^0 + q}{n_{ij} + n_{ij}^0 + q + 1}\right)\right] - 2\prod_{j=1}^{k_i} \times \left[\prod_{q=1}^{k_{ij}}\left(\frac{n_{ij} + n_{ij}^0 - s_{ij} - s_{ij}^0 + q}{n_{ij} + n_{ij}^0 + q + 1}\right)\right] + 1, \quad \text{for a parallel subsystem.} \tag{5}$$

Note that M_i and W_i given in (4) and (5) are the first two moments, $E(r_i)$ and $E(r_i^2)$, of (1) and (2) for a series and parallel subsystem configuration, respectively. The corresponding approximate induced prior beta cdf on r_i is denoted by $G_a(r_i)$.

As discussed by MWF (sec. 2.3), in certain cases techniques exist for obtaining the exact induced subsystem prior distribution $G(r_i)$ in closed form. Computational instabilities often occur when using these exact methods, however. Consequently, as in MWF, we examine the quality of the beta approximation to the exact induced ith subsystem prior by simulating the distribution of r_i defined by either (1) or (2) and by computing the corresponding Kolmogorov–Smirnov (KS) two-sided p value for the beta hypothesis.

MWF described and used a method first proposed by Winkler (1968) for averaging the ith subsystem native and induced priors to produce a single combined beta prior. This same method is used here. Further, the MWF requirement that s_i^0 and n_i^0 be nonnegative integers is relaxed here.

We now use the Bayes theorem in conjunction with the binomial test data and the combined beta prior to obtain the ith subsystem posterior beta distribution. The details are found in MWF. Recall that the mth subsystem denotes the overall system. Consequently, (s_m, n_m) denotes binomial system test data, and the corresponding native prior parameters (s_m^0, n_m^0) also denote system beta prior parameters.

As discussed earlier, the ith subsystem posterior beta distribution given previously may serve as a component beta prior distribution in a subsequent higher-order subsystem. Denote this higher-order subsystem by l and the component, which the ith subsystem occupies, as p. In such a case, the corresponding component beta prior parameters (s_{lp}^0, n_{lp}^0) for use in the lth subsystem analysis are given by

$$s_{lp}^0 = w_{i1}(a_i + w_{i2}s_i^0) + s_i + w_{i2} - 1$$

$$n_{lp}^0 = w_{i1}(a_i + b_i) + w_{i2}n_i^0 + n_i + 2w_{i2} - 2. \quad (6)$$

3. EXAMPLE

As discussed in Section 1, the example concerns the unavailability on demand due to standby failure of the LPCI system in a certain U.S. commercial boiling-water reactor. Figure 1 shows the demand-availability block diagram for this system. Because the Bayesian model presented here is based on reliability rather than unreliability considerations, we will consider availability on demand throughout the analysis and express only the final results and estimates in terms of unavailability.

Table 1. Accident Sequence Evaluation Program LPCI System-Component Beta-Prior Demand-Availability Data

Component	Failure mode	s_{ij}^0	n_{ij}^0
Motor-driven pump	Failure to start	190.17	189.79
Motor-operated valve	Failure to operate	469.13	469.90
Check valve	Failure to open	14,231.34	14,232.12

Three data sources are considered. Table 1 contains component-demand-availability data in the form of beta distribution parameters on motor-driven pumps, motor-operated valves, and check valves based on data reported in the Nuclear Regulatory Commission Accident Sequence Evaluation Program data base (U.S. Nuclear Regulatory Commission 1987). These data represent a compendium of several data sources for similar components in similar nuclear-power-plant systems; thus these data are appropriate for use in a Bayesian analysis as prior-component data. The data were fitted to beta priors by matching either moments or quantiles. It is observed that the pump data exhibit the greatest uncertainty, whereas the check-valve data express the largest mean availability. Although $s_{ij}^0 > n_{ij}^0$ for motor-driven pumps in Table 1, the interpretation is correct and follows that of MWF.

Table 2 contains the binomial-component-test data for each of the components in the coolant injection system in the specific plant of interest, here based on monthly testing of each component. The data were obtained from the plant testing and maintenance records. Each component is tested separately, and, consequently, each component is considered to have a uniquely different underlying availability value. Thus all components are considered to be nonidentical throughout the analysis, and $k_{ij} = 1$ for all i and j. Several components are replicate designs, however; pumps A and D; pumps B and C; CV-48 A, B, C, and D; MOV-25 A and B; and CV-46 A and B are the same. In Table 2, s_{ij} represents the number of nonoccurrences of the corresponding failure mode when $n_{ij} = 240$ test demands.

Table 3 presents demand-availability data in the

Table 2. Plant-Specific LPCI System-Component Binomial Test Data

Component	Failure mode	s_{ij}	n_{ij}
Pump A	Failure to start	236	240
Pump B	Failure to start	240	240
Pump C, D	Failure to start	238	240
CV-48 A, B, C, D[a]	Failure to open	240	240
CV-46 A, B[a]	Failure to open	240	240
MOV-25 A, B[b]	Failure to operate	240	240

[a] CV indicates check valve.
[b] MOV indicates motor-operated valve.

Table 3. IEEE Std. 500 LPCI Subsystem Beta Prior Demand-Availability Data

Subsystem	*Failure mode*	s_i^0	n_i^0
Pump train A, B, C, D	Failure on demand	.55	−.42
LPCI subsystem A, B	Failure on demand	241.87	242.66

form of beta distribution parameters on the lowest-order subsystems in the system—namely, pump trains A, B, C, and D and LPCI subsystems A and B. These data are based on composited IEEE Std. 500 reliability data (IEEE 1983) from sections 11.1.2.4.2.2, 11.2.3, and 11.2.a.2. Because the data are a composite of many sources, they will be used as subsystem-level prior data in the analysis. The pump-train beta parameters in Table 3 represent a diffuse beta prior. This is a direct consequence of the large difference between the recommended and high failure-rate estimates for a centrifugal residual heat-removal pump given in the IEEE Std. 500 data. There exist neither test nor prior data on the higher-order subsystems in the system in Figure 1; thus only the data in Tables 1–3 will be used in the analysis.

From the data in Tables 1–3, we have the exact same state of knowledge for subsystems 1 and 2 regarding their availability, although they are nonidentical. The same situation exists for subsystems 4 and 6. Thus there is no need to analyze such pairs separately. Effectively then, there are only nine different subsystems that must be analyzed in the system.

Consider LPCI subsystem A (or B). This is a series subsystem of two components ($k_1 = 2$). The induced prior approximate distribution for the availability of LPCI subsystem A on demand is computed according to (3) to be a $\beta(743.41, 1.94)$ distribution.

Now let us compute the combined prior. The native prior IEEE data in Table 3 is probably based on some of the same data sources as the data in Table 1. Accordingly, the sum of the weights was taken to be 1. To avoid placing too much weight on the IEEE data relative to the two other data sources, weights of .75 and .25 were placed on the induced and native priors, respectively; the sensitivity of the results to this choice will be considered later, however. This choice yielded a $\beta(618.27, 1.91)$ combined prior. Because there are no binomial test data on LPCI subsystem A, the posterior reduces to the combined prior.

Now consider pump train A (subsystem 3). The induced prior approximate distribution for this series subsystem availability is computed to be a $\beta(431.89, 4.72)$ distribution. The same weights were chosen as before, yielding a combined $\beta(324.31, 3.55)$ prior (and posterior) distribution.

As mentioned earlier, pump trains C and D (subsystems 4 and 6) have the same state of knowledge regarding their availability. The induced prior for each series subsystem availability is computed to be a $\beta(437.57, 2.73)$ distribution. With the same weights as before, the combined prior (and posterior) is a $\beta(328.57, 2.05)$ distribution. Pump trains C and D have an availability distribution that is located just slightly to the right of the corresponding distribution for pump train A because pump A was observed to have two more failures to start on demand than pumps C or D.

Similarly, the induced prior approximate distribution for pump train B (subsystem 5) availability is computed to be a $\beta(466.93, .73)$ distribution. Again, using the same weights, the combined prior (and posterior) is a $\beta(350.59, .55)$ distribution. Note that the availability distribution for pump train B is shifted to the right of pump trains C and D.

Now consider pump subsystem A (subsystem 7). This subsystem is different in that subsystems 3 and 4 become the two components in this parallel subsystem. From (6), the beta posterior parameters for subsystem 3 correspond to component beta prior parameters $s_{71}^0 = 323.31$ and $n_{71}^0 = 325.86$. Similarly, from (6) the component beta-prior parameters for subsystem 4 are $s_{72}^0 = 327.57$ and $n_{72}^0 = 328.62$. As previously discussed, there are no binomial-test data for these two components and, again using (3), the induced-prior approximate distribution for the demand availability of pump subsystem A is computed to be a $\beta(16{,}625.22, 1.12)$ distribution.

The calculations for the remaining subsystems are quite similar and equally straightforward. Table 4 summarizes the results for all 11 subsystems and includes the two-sided KS p values for the hypothesis of an approximate induced-beta-prior distribution, the parameters of the induced beta prior, and corresponding posterior distributions, as well as the .05, .50, and .95 quantiles of the posterior distribution.

One important practical benefit of using this method is apparent in Table 4. Because of the bottom-up system decomposition, the intermediate subsystem results track the increase or decrease in availability as the system increases in complexity. For example, there is roughly an increase of .008 in the median availability of two pump trains in parallel (pump subsystem A) over a single train. Such results are helpful in understanding and ordering the contributors to final system availability.

Finally, we see from Table 4 that the required posterior distribution of unavailability on demand for this system is a right-skewed (L-shaped) $\beta(.78, 80{,}745.70)$ distribution. A common Bayesian point estimate of the demand unavailability of this system

Table 4. Subsystem Results for the Demand Availability of the LPCI System

Subsystem	Name	Induced beta prior	KS p value	Beta posterior	Posterior quantiles .05	.5	.95
1	LPCI subsystem A	$\beta(743.41, 1.94)$	.50	$\beta(618.27, 1.91)$	.9926	.9974	.99949
2	LPCI subsystem B	$\beta(743.41, 1.94)$	.50	$\beta(618.27, 1.91)$	.9926	.9974	.99949
3	Pump train A	$\beta(431.89, 4.72)$	.99	$\beta(324.31, 3.55)$	.978	.9901	.9966
4	Pump train C	$\beta(437.57, 2.73)$	.55	$\beta(328.57, 2.05)$	.985	.9948	.9989
5	Pump train B	$\beta(466.93, .73)$	.30	$\beta(350.59, .55)$	.9942	.99922	.999990
6	Pump train D	$\beta(437.57, 2.73)$	.55	$\beta(328.57, 2.05)$	.985	.9948	.9989
7	Pump subsystem A	$\beta(16{,}625.22, 1.12)$	.49	$\beta(16{,}625.22, 1.12)$	.99981	.999951	.9999955
8	Pump subsystem B	$\beta(32{,}444.46, .32)$	.37	$\beta((32{,}444.46, .32)$	.999956	.9999974	.9999999980
9	LPCI train A	$\beta(631.26, 1.99)$	.74	$\beta(631.26, 1.99)$	.9925	.9974	.99944
10	LPCI train B	$\beta(620.20, 1.92)$	.58	$\beta(620.20, 1.92)$	.9926	.9974	.99948
11	LPCI system	$\beta(80{,}745.70, .78)$	.21	$\beta(80{,}745.70, .78)$	.999968	.9999940	.99999975

is the posterior mean of 9.7×10^{-6}, but a two-sided symmetric 90% Bayesian probability interval for the unknown unavailability is $(2.5 \times 10^{-7}, 3.2 \times 10^{-5})$. The posterior median is 6.0×10^{-6}. From the posterior mean, it is estimated that the LPCI system will be unavailable on demand (as a consequence of a standby failure) an average of once every 103,000 demands.

Let us now examine the sensitivity of the results to the data sources and weights used in the analysis. Such an analysis is useful in understanding the influence and contribution of the various data sources to the results, and we recommend it as the final step when using this method. Table 5 gives the final posterior system-demand unavailability distribution for several choices of weights applied to the IEEE data and for all possible combinations of the three data sources used in the analysis. The number in parentheses in the column labeled "IEEE" is the weight w_{i2} applied to the native prior when computing the combined subsystem prior. The absence of IEEE data corresponds to $w_{i2} = 0$ and the absence of both other data sources corresponds to $w_{i2} = 1$.

When binomial test data are used without the Accident Sequence Evaluation Program data, some kind of noninformative component prior must be used. In cases 3 and 5, Jeffreys's noninformative $\beta(.5, .5)$ priors were used, and $\beta(1, 1)$ (uniform) priors were used in case 6. By comparing cases 5 and 6, we see that the use of uniform priors tends to shift the unavailability distribution toward higher unavailability values. The use of Jeffreys's priors also produces a somewhat more diffuse system posterior distribution. Since $\beta(.5, .5)$ is more diffuse than $\beta(1, 1)$, the component posteriors tend more toward the high component availabilities implied by the test data when using Jeffreys's priors. For this reason we recommend the use of Jeffreys's noninformative priors when few (if any) component failures have been observed.

Cases 1, 2, 8, and 9 illustrate the resulting effect on the system posterior-demand-unavailability dis-

Table 5. Sensitivity of the Posterior LPCI System-Demand-Unavailability Distribution to the Data Sources and Weights Used in the Analysis

Case	Data source: Test	ASEP[c]	IEEE	Posterior distribution	Mean	Quantiles: .05	.50	.95
1	yes	yes	yes(.25)	$\beta(.78, 80{,}745.70)$	9.7E-6	2.5E-7	6.0E-6	3.2E-5
2	yes	yes	no(0)	$\beta(.80, 114{,}764.31)$	7.0E-6	2.0E-7	4.4E-6	2.3E-5
3[a]	yes	no	yes(.25)	$\beta(.46, 17{,}738.31)$	2.6E-5	6.6E-8	1.1E-5	1.0E-4
4	no	yes	yes(.25)	$\beta(.75, 38{,}837.29)$	1.9E-5	4.2E-7	1.2E-5	6.4E-5
5[a]	yes	no	no(0)	$\beta(.37, 20{,}038.40)$	1.8E-5	1.1E-8	6.1E-6	7.9E-5
6[b]	yes	no	no(0)	$\beta(.88, 12{,}067.35)$	7.3E-5	2.6E-6	4.7E-5	2.3E-4
7	no	yes	no(0)	$\beta(.76, 50{,}100.78)$	1.5E-5	3.5E-7	9.2E-6	5.0E-5
8	no	no	yes(1)	$\beta(.20, 3{,}366.00)$	5.9E-5	5.3E-11	6.0E-6	3.0E-4
9	yes	yes	yes(.5)	$\beta(.76, 52{,}308.11)$	1.5E-5	3.4E-7	8.9E-6	4.8E-5

[a] Jeffreys's noninformative component priors.
[b] Uniform component priors.
[c] Accident Sequence Evaluation Program.

tribution as the weight given to the IEEE data goes from 0 to 1. The mean system unavailability increases by almost an order of magnitude, reflecting the somewhat higher component mean unavailabilities of the IEEE data. As w_{i2} increases from 0 to .5 in the presence of both other data sources (cases 2, 1, and 9), the system distribution shifts slightly toward higher unavailability values, but the dispersion remains relatively unchanged. The value chosen for w_{i2} thus has relatively little effect, provided the test and Accident Sequence data are used.

Cases 2, 5, 7, and 8 compare the effects of the IEEE data to those produced by either (or both) of the other two data sources. Because of the diffuse IEEE pump-train data, the IEEE data by itself produces a much more diffuse system-posterior distribution than either or both of the other sources. Case 8 represents the most diffuse posterior distribution present.

Cases 5, 7, and 8 compare the results when using only a single data source. In the absence of the Accident Sequence data, both the test and IEEE data-produced posteriors are quite diffuse with the IEEE data producing a significantly more diffuse posterior (cases 5 and 8). Cases 5 and 7 compare the results for test versus Accident Sequence data in the absence of IEEE data. The results using test data only are slightly weaker (more diffuse) than using only Accident Sequence data, but both posteriors have about the same location. Observe that the Accident Sequence data produce a corresponding posterior that is the least diffuse of all three (case 7). In this sense, the Accident Sequence data have the most influence on the final system posterior. This can be clearly seen in Figure 2, in which all nine system posterior demand-unavailability probability-density functions are plotted. The upper three density functions correspond to the absence of Accident Sequence data (cases 3, 5, and 8), and the second group (cases 1, 2, 4, 7, and 9) all have this source present. All of the densities are L-shaped beta distributions, and a log scale for unavailability is used to separate the densities. Three distinct groupings are readily apparent. Case 6, the use of only test data in conjunction with uniform component priors, is also clearly in a separate class. In order of diffuseness, the upper set of three densities is most diffuse (because of the sharper "bend" in the L-shaped density), followed by the second set of five densities. Finally, case 6 is the least diffuse.

The use of uniform component priors in the absence of Accident Sequence and IEEE data reduces posterior diffuseness even more than the additional use of Accident Sequence data (see cases 2, 5, and 6 in Fig. 2). Thus, in the absence of Accident Sequence data, the choice of noninformative component priors can be quite influential, especially on the interval estimation of system demand unavailability. It is clear that the use of uniform component priors may be producing some degree of spurious variance reduction in this example. Consequently, as stated earlier, we recommend the use of Jeffreys's component priors.

Cases 2 and 5 show the effect of adding the Accident Sequence data to an analysis in which only the test data is used in the absence of IEEE data. The net effect is to shift the final posterior slightly toward smaller unavailability values and to decrease the diffuseness as a consequence of the use of the Accident Sequence data.

4. DISCUSSION

A Bayesian procedure for determining the reliability (or availability) of a complex system of independent binomial series or parallel subsystems has been developed. The procedure uses both test and prior data at the component, subsystem, and system levels. Although the procedure is a Bayesian one based on subjective degree of belief in the form of beta prior distributions, an analysis using only binomial test data may be performed. Noninformative component priors would be assigned, and the resulting posterior system distribution could then be used to provide estimates that depend almost exclusively on the test results. Some degree of subjectivity still remains, however, as a consequence of the choice of noninformative priors; cases 5 and 6 in Table 5 illustrate this.

Existing Bayesian-system reliability methods are often inadequate for several reasons. Most existing methods are either mathematically or computationally cumbersome and are difficult to implement in practice. The required posterior distributions are often quite complicated. In contrast, this method is easily implemented, and the required posterior distributions are usually well approximated by beta distributions. Many methods consider only point estimates, but this method considers both point and interval estimates that are easily computed. Unlike other methods, this method provides clear insight regarding the contributors and makeup of the estimated reliability of a complex system (including the uncertainty in the reliability estimate) because of its iterative bottom-up structure. We have found this to be extremely useful in practice. Sensitivities of the system reliability to the prior and/or test data can be easily examined. Most existing methods that consider both system and component prior information use complex top-down inductive arguments to apportion the system-level prior distribution at the component level so as to ensure consistent prior be-

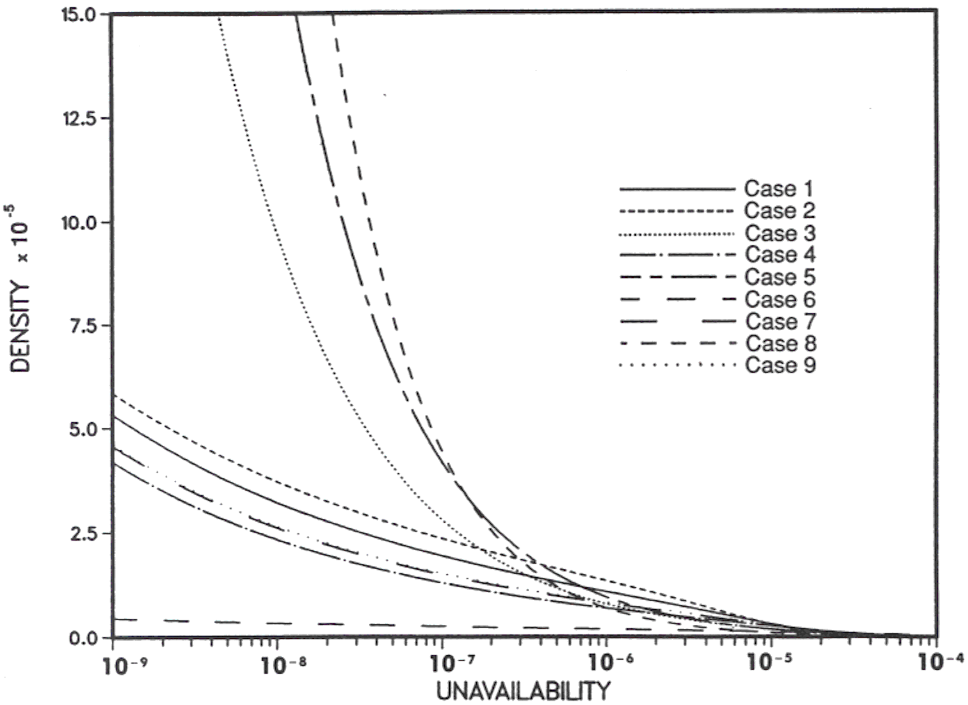

Figure 2. The Posterior LPCI System-Demand-Unavailability Distributions in Table 5.

liefs. The method presented here resolves such inconsistencies in a straightforward deductive way as simple averages of inconsistent prior distributions. Several existing methods consider both component and system-level tests and/or prior data but in a much more restrictive manner than the method considered here. In fact, we know of no other Bayesian method that integrates component, subsystem, and system prior and test data in the unified manner considered here.

There is an important analogy of this Bayesian procedure in fault-tree analysis. Test and/or prior data can be incorporated in fault-tree models, not only on the elementary basic events in the tree but also on events above the basic-event level, even for the top event itself. The main limitation concerns the location of any identical basic events in the tree and the restriction required here that such repeated events occur only within the same substructure. This requirement essentially restricts the use of this method to rather simple fault-tree models.

The method also provides a convenient means to assess the impact on system availability of test data at various levels within the system. This cannot be done directly with existing models that ignore multilevel data. For example, the benefits to system availability of additional component versus subsystem-level test data can be readily and quickly determined. Thus the Bayesian model presented here can be a useful aid in allocating test resources at any level from component through complete-system testing.

Finally, we have developed a basic FORTRAN 77 interactive computer program to implement this procedure. The program is named BAPSS (*B*ayesian *A*nalysis of *P*arallel/*S*eries *S*ystems). A listing of this program and sample output for the coolant-injection-system example considered here are available from us on request.

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