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Title:

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Characterizing Reliability During a Product Development Program

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ABSTRACT: Delphi Automotive Systems and the Los Alamos National Laboratory worked together to develop a methodology to characterize the reliability of a new product during its development program. One of the traditional techniques utilized in this regard is Reliability Growth Testing (RGT). This paper outlines a more timely and efficient approach to achieving and maintaining the reliability perspective. Rather than conducting testing after hardware has been built, and developing statistical confidence bands around the results, this updating approach starts with an early reliability estimate characterized by large uncertainty, and then proceeds to reduce the uncertainty by folding in fresh information in a Bayesian framework. A considerable amount of knowledge is available at the beginning of a program in the form of expert judgment which helps to provide the initial estimate. This estimate is then continually updated as substantial and varied information becomes available during the course of the development program.

1 SUMMARY AND CONCLUSIONS

A methodology has been developed to characterize the reliability of new product programs during their development phase. This approach has been found to be effective in evaluating systems ranging from automotive to national defense. Just as estimates of cost and program timing are critical factors to be known and monitored during program development, product reliability also needs to be addressed. The reliability estimate and the uncertainty of that estimate are an excellent way to provide this characterization. The methodology involves combining information that ranges from qualitative, such as expert judgment, to quantitative, such as test data. It is possible to develop realistic reliability estimates at the beginning of a new product program, even though hardware is not available, because a considerable amount of knowledge exists in the experience base of engineers. This knowledge is elicited in the form of expert judgment. The process of estimating the reliability characterization proceeds during the entire development program, incorporating information from any available source (i.e. supplier, customer), about any level of the product (i.e. subsystem, component). The approach allows the reliability of the new product to be characterized early, before hardware exists, and to be updated as the design evolves. This allows the project team to "keep score" as they work through the program to design in reliability. The results may also be used to provide steerage to the project team with regard to how to drive reliability higher and / or reduce the uncertainty in reliability.

The challenge has been to develop a framework for this reliability characterization which is physically, logically, and mathematically sound, but which is flexible enough to accommodate all of the diverse information that becomes available, and responsive enough to provide timely results which support the development process. The information updating approaches (such as those based on Bayes Theorem) are suggested as key methods directly applicable to this problem. This paper describes an approach to reliability modeling that encompasses the impact of both product and manufacturing process design on the distribution of reliability over time. Such a distribution represents the uncertainty associated with the reliability at any given time. This work builds on methods previously published by the same authors (Kerscher et al. 1998). The approach in both papers describes the elicitation and analysis of expert judgment which is used to quantify the initial reliability estimate, including uncertainty. The approach also describes Bayesian updating which is applicable throughout the development program, and which accommodates a wide variety of possible new information sources. Although the model is rigorous in its execution, some user friendly approximations are also described which may be useful to the product development team for purposes of test and validation planning. The whole idea is to allow new project development teams to address the reliability issue with the same focus that they traditionally have on cost and timing.

2 INTRODUCTION

Over the years many advancing techniques in the area of reliability engineering have surfaced. One of these techniques in the military sphere of influence is Reliability Growth Testing (RGT). Private industry has reviewed RGT as part of the solution to their reliability concerns, but many practical considerations have slowed its implementation. It's objective is to demonstrate the reliability requirement of a new product with a specified confidence. This paper speaks directly to that objective but discusses a somewhat different approach to achieving it. Rather than conducting testing as a continuum and developing statistical confidence bands around the results, this approach starts with a reliability estimate characterized by combining all available information and data sources at the time. Because this initially results in revealing large uncertainties, it then proceeds to reduce the uncertainties by folding in fresh information.

In the traditional military context a product would be developed (or an existing product modified), and then the product would be put on test. The typically long-term test was designed to statistically demonstrate a reliability requirement at a specified confidence. This product was then delivered to the military services with demonstrated reliability as part of the deliverables package. The fact that the test involved additional time, cost and resources was deemed to be acceptable. In the industrial setting, however, these drawbacks can become acute, and in many cases deter the use of this traditional approach. Also, although not planned, it is possible for the end of a development program to approach the scheduled start of volume production. RGT at this point is seen not only as an additional amount of time and expense in the development program, but also as a holding item before production may begin.

Probably the most significant negative factor, however, has nothing to do with timing and resources, but rather the organizational environment that design engineers are asked to work within. Not atypically, the reliability growth test may be the first large-scale organized development test to be conducted on the new product design. In some applications such as nuclear weapons, such large-scale tests are prohibited by treaty. The results typically identify several weak spots / failures in the design, which should be expected. The reliability growth test, however, has been organized to demonstrate the desired reliability, and do it efficiently, by organizing the test around an anticipated few or no failures. The result is a triple blow to the design program. First, it demonstrates that the desired reliability has not been achieved. Second, it demonstrates it with statistical confidence, and finally, it may produce this result near to the scheduled start of volume production, which dictates the choice of shipping defective product or delaying the start of production. Perceptive program managers who recognize the deficiency of their product in the area of reliability naturally tend to resist demonstrating the fact without sufficient time to respond. All of these factors tend to work against the implementation of traditional RGT in an industrial setting.

There is a definite need, however, for an understanding of the reliability perspective of a new product during its development program. Identifying the uncertainty in the reliability estimates, which typically drives the unreliability, and doing it early enough in the development cycle for corrective action to be organized by the development team, has been found to be a culturally acceptable way to approach the reliability issue, and can therefore be a powerful factor in the drive for high reliability. The information combination and updating approach is a methodology which is directly applicable to this problem.

The following notations are used :

R_i	reliability characterization of a system , estimated at time step, i.
$f(R_i)$	probability distribution function of R_i, representing the uncertainty in system reliability.
λ	failure rate for a component, subsystem or system (e.g., failures per vehicle per scaled unit of time) and scale parameter of the Weibull distribution.
t	time.
β	slope or shape parameter of the Weibull distribution.
R(t)	reliability from a two-parameter Weibull distribution.
$\Gamma(n)$	gamma function, which is the $\int x^{(n-1)} e^{-x}\,dx$ from 0 to 1.
θ	parameter of interest.
(a,b)	two parameters of the beta distribution, sometimes referred to as the pseudo successes and pseudo failures, respectively.
p	probability of success of a trial.
n	number of tests.
(α, η)	two parameters of the gamma distribution, sometimes referred to as the pseudo failures and pseudo total transformed test time, respectively.
s	failures.
τ	total transformed test time $(t_1^{\beta} + t_2^{\beta} + \ldots)$

3 OVERVIEW OF RELIABILITY UPDATING METHODOLOGY

The reliability of the product (including the manufacturing process) at any given point in time or at any given step in the overall product / process design assurance program is what has been referred to by the term reliability characterization. "Reliability characterization" includes both the functional calculation of the reliability (point estimate value) and the uncertainty (usually represented by a distribution function) that accompanies the reliability value. Reliability values can be calculated from formulas or models, which integrate the structure of the system. For purposes here, the system structure is represented by a reliability block diagram.

Either the reliability calculation and / or its uncertainty distribution can change due to various changes in the development program (Hulting et al. 1994). Examples include the development of expert judgment from changes in the existing state of knowledge, the determination of requirements, the availability of test data or supplier information, the implementation of corrective actions, etc. New components or failure modes may be added, or existing elements deleted, as the design evolves. Changes can occur in both the product design as well as the manufacturing process which can affect the reliability value and / or its associated uncertainty.

Once a change occurs anywhere in the development process, a new step (i) is designated and a new reliability, R_i, is calculated along with a new uncertainty distribution, $f(R_i)$. The tracking of these reliability snapshots over time is one method of monitoring how the changes in reliability are approaching the target value, as part of the validation effort.

At each reliability snapshot, gaps in the current state of knowledge become apparent, providing the project team with a rational basis for a strategy for deciding where to devote future testing and analysis resources (i.e. a reliability growth plan). In a proactive sense, "what if" analyses allow the project team to develop the optimal test / analysis and validation plan given existing constraints of hardware, facilities, timing, etc. The power of these "what if" approximations lies in gaining understanding about the <u>potential</u> impact of the test / analysis, and allowing the project team to judge the usefulness of the effort <u>before</u> it is started. The existence of the reliability characterization also allows the customer to participate in a constructive way, if desired, and also provides an avenue for suppliers to contribute, if appropriate. This methodology was evaluated on a program in the automotive industry, the results of which are the subject of this paper and the previous paper discussed earlier (Kerscher et al. 1998).

4 FRAMEWORK

One of the first activities of an organized reliability program is the construction of a reliability logic flow diagram (e.g. reliability block diagram, success tree) representing the structure of the product under development. The framework of the reliability characterization involves selecting a mathematical model following the logic flow in that diagram, making an initial estimate of the parameters identified in the model, and organizing a methodology for updating the model as new information becomes available. Section 5 describes the Weibull functions selected to model the product reliability. Section 6 describes the elicitation of expert judgment which is used to develop the initial (or prior information-based) estimate of the model parameters. Section 7 describes the use of Bayes Theorem to update the model. Also, Section 8 describes some useful approximations that may be used for planning purposes.

5 DESCRIPTION OF WEIBULL MODELS

The concept of the hazard function of a manufactured product being made up of definable portions such as infant mortality, useful life, and wearout, has long been utilized (Kerscher 1989). It is further suggested here that the "infant mortality" is mainly due to the latent defect subpopulation generated during the manufacturing process, and the "useful life" portion is primarily due to latent design defects which manifest themselves over the life of the product. "Wearout" is the third sub-population of parts which fail due to failure modes associated with operating the product beyond its useful life. Good engineering practice has long held that wearout failure modes should be identified during the development process, and that those failure modes that cannot be designed out should at least be designed to occur beyond the useful life of the product. For the purposes of this paper any wearout failure modes are assumed to occur beyond useful life, and are not, therefore discussed here. The approach to identifying and addressing the latent defects in the first two sub-populations is not as well established, although that is in fact the objective of a comprehensive design assurance program (Kerscher 1993). A first helpful step in identifying those latent defects is the establishment of a reliability model. Figure 1 shows a portion of the reliability logic flow diagram used in the automotive program. The section shown is in the form of a success tree diagram. The two parameter Weibull may be used to model both the defect subpopulation due to the manufacturing process, as well as the defect sub-population due to the product design (Kerscher 1989). The total distribution is the combination of the two sub-populations.

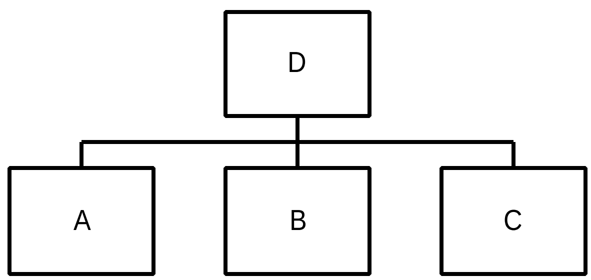

Figure 1. Reliability Success Tree Diagram

The two-parameter Weibull expression for reliability is given in equation (1).

$$R(t) = \exp(- l\ (t)^{b}) \tag{1}$$

This version of the Weibull separates the two parameters and often simplifies the algebra and the subsequent Bayesian manipulations (Martz et al. 1982). The challenge is to identify the two parameters; b (the slope) and l (the failure rate per scaled unit of time) (Martz et al. 1982). Section 6 describes the elicitation of expert judgment to provide initial estimates of these parameters. The approach for this specific example is detailed at length in Kerscher et al. (1998). Table 1 lists the two parameters, b and l, of both the manufacturing and design defect sub-populations, for the components in the example: A, B, and C. No information was elicited for the subsystem D, whose reliability is defined by the logic flow diagram (Fig. 1) and the reliabilities of components A, B, and C.

Once the individual distributions for the latent design and manufacturing defects have been identified, they may be combined to produce the distribution representative of the whole component or subsystem. All of the individual distributions of the individual elements may then be combined according to the reliability logic flow diagram to form the distribution representative of the entire product. Estimates of reliability (including uncertainty) can then be calculated using eq (1) at various points in time for predicting the long term performance.

As part of the logic diagram, how the blocks interact / connect is specified as are any levels within the blocks (e.g., component, subsystem and system). These interrelations of the blocks will determine how the reliability is to be calculated at various levels. For instance, if the components within a block (A, B, and C in the example in Fig. 1) are all in series, the block (subsystem D) reliability is the product of the reliabilities of the components.

Table 1. Weibull Parameters for Design and Manufacturing Models and Initial Reliability Estimates at 12 Months and 100,000 Miles

	Parameters				Reliability R0					
	Design		Manufacturing		12 Month			100,000 Miles		
					5	50	95	5	50	95
	b	*l*	*b*	*l*,	Percentiles					
Component A	0.75	0.00001	0.14	5.17	0.9996	0.9999	1	0.9993	0.9999	1
Component B	0.75	0.00002	0.43	9.94	0.9989	1	1	0.9986	0.9999	1
Component C	0.75	0.001	0.42	4.18	0.976	0.9989	0.9999	0.8829	0.9952	0.9997
Subsystem D	~	~	~	~	0.9723	0.9985	0.9998	0.8794	0.9944	0.9994

6 ELICITATION OF EXPERT JUDGMENT AND INITIAL RELIABILITY CHARACTERIZATION

To obtain an initial overall reliability estimate, R_0, of the entire logic flow diagram, estimates of component and subsystem reliability's (with uncertainties) were elicited from teams of subject matter experts. The experts had been previously selected by their managers and peers as being knowledgeable of their subsystem or component. The elicitations were first conducted with those working on the product design and then with those working on the manufacturing process.

The experts were not asked to estimate reliabilities, per se, but allowed to provide their estimates about component, subsystem and system performance in terms familiar to them. (This approach and its benefits are described in further detail in Meyer et al. (1991)). For example, the experts in the design process gave their estimates as incidents per thousand vehicles (IPTV), while those familiar with the manufacturing process gave their estimates as parts per million (PPM). As part of their estimates, the experts were asked to give a very brief explanation of their reasoning. In addition, the experts provided ranges on their estimates, which were used to represent the uncertainty and ultimately formulate $f(R_i)$.

It should be noted that information about failure modes of various blocks, and their apportionment, can also be elicited during the initial characterization. This may become important later when tests are planned or performed on a subset of failure modes.

The results from the design elicitations were presented to all of the participating experts for their review and reconciliation across the entire system. This information was then used to calculate the *b* and *l* parameters for design and manufacturing as given in Section 5. The uncertainty expressed in the expert elicitations was transformed into distributional information in the mathematical model.

Reliabilities were then calculated using eq (1), with subsystem and system estimates being calculated using the reliability logic flow diagram and numerical sampling techniques. The results included reliabilities in distributional form (reflecting the uncertainty) for components, subsystems and the system at various times. The results for the initial reliability characterization, R_0, at 12 months and 100,000 miles are summarized in Table 1. For instance, the median reliability of subsystem D at 12 months was estimated to be 0.9985, with the 5 th and 95 th percentile reliability estimated at 0.9723 and 0.9998 respectively.

Subsequent information, including new test data, is reflected in subsequent values of R_i and $f(R_i)$ as described in Section 7. In this way reliability may be monitored over time (reliability growth), and plans formulated accordingly.

7 DESCRIPTION OF UPDATING METHODOLOGY

Pooling data from different sources or of different types (e.g. tests, process capability studies, engineering judgment) is usually done with methods that combine the distribution functions associated with the various information sources. Bayes Theorem offers one mechanism for such combination. Bayesian pooling combines information with the following structure: the existing information (data) forms a distribution, called the likelihood. That likelihood distribution is formed from the data / information symbolized by the random variable, x, and it has characteristics (i.e. parameters), such as a mean. That parameter(s) is not considered a fixed quantity but instead, has its own probability distribution, called the prior. The prior is combined with the likelihood using Bayes Theorem to form the resulting or posterior distribution. Bayes Theorem is used to calculate the posterior distribution, $g(q|x)$, from the likelihood distribution, $f(x|q)$ as:

$$g(q|x) = [f(x|q)\, g(q)] / \int f(x|q)\, g(q)\, dq \qquad (2)$$

where $g(q)$ is the prior distribution on the parameter of interest, q. Bayesian combination is often referred to as an updating process, where new information is combined with existing information.

Simulation methods are often used to combine or propagate uncertainties (represented as distribution functions) through the logic flow diagram, as well as accomplishing the Bayesian combination itself. This is the approach taken with this project. The range and nominal estimates provided through the expert elicitation are used to form empirical distribution functions for reliability (initial reliability characterization) for each item in the logic flow diagram. Monte Carlo simulation is used to propagate reliability characterizations through the various levels of the diagram, with the accuracy being dependent on the number of simulations. The posterior distributions resulting from the simulation are empirical in form, meaning they are not fit to any particular distribution (e.g., a beta) or distribution family. It is not necessary to develop prior information for subsystems above the component level. These are available by combining the reliability characterizations from the levels below. However, if there is information on these subsystems, the reliability characterization from that information can be combined with the distribution from levels below using methods in Martz et al. (1997, 1990, 1988). More importantly, test data and other new information can also be added to the existing reliability characterization at any level and / or block (e.g. system, subsystem, component). This data may be applicable to the entire block, or only to a single failure mode within the block. This process is presented in detail in Martz et al. (1988) for series systems and in Martz et al. (1990) for series / parallel systems.

In general, the initial reliability characterization R_0, is developed from expert judgment and is referred to as the native prior distribution. During the course of the development program data may be developed regarding each element (e.g. system, subsystem, component) and this would be used to form data (or likelihood) distributions. All of the distribution information in the items at the various levels must be combined up through the logic flow diagram, to produce a final estimate of the reliability and its uncertainty at the top, or system, level. Three different combination methods are used:

- For each prior distribution that needs combining with a data distribution (in a block), Bayes Theorem is used and a posterior distribution results.
- Posterior distributions within a given level are combined according to the logic of the logic flow diagram to form the induced prior distribution of the next higher level.
- Induced prior and native prior distributions at the higher levels are combined within the same item using a method in Martz et al. (1988) to form the combined prior (for that block) which is then merged with the data (for that block) via method 1. This approach is continued up the diagram until a posterior distribution is developed at the system level.

As more data becomes available and incorporated into the reliability characterization through the Bayesian updating process, this data will tend to dominate over the effects of the initial estimate developed through expert judgment. In other words, R_t formulated from many test results will look less and less like R_0 from expert estimates. It should be noted that updating can be done by combination methods other than Bayes Theorem (Meyer et al. 1991).

A single update from our example will be helpful to illustrate. Figure 2 shows the probability distributions of reliability at 12 months for the components and subsystem in the example at a certain point during the development program. Note that there is considerable uncertainty around component C which is reflected in subsystem D (note also the difference in x-axis scales). In our example, 60 samples of component C were tested for 12 months with no observed failures, and this was treated as an update event.

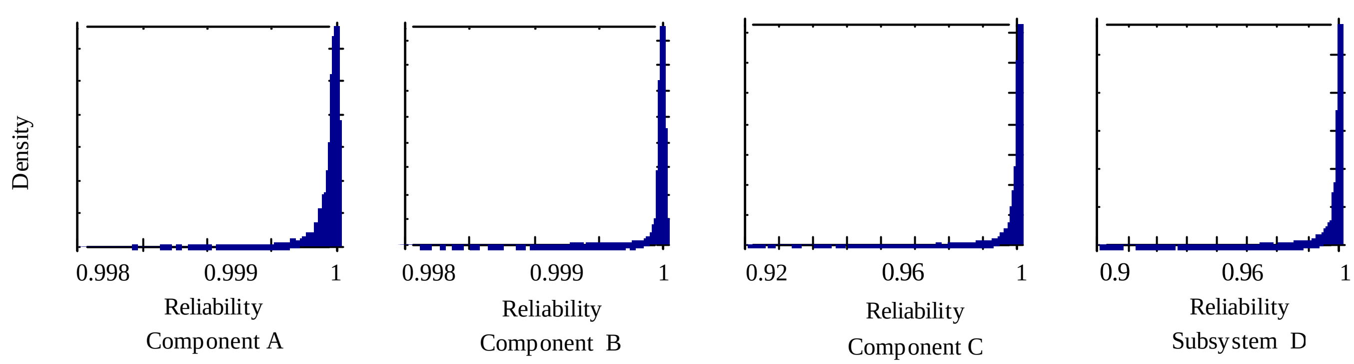

Figure 2. Reliability Prior Distributions @ 12 Months

Figure 3 shows this data and the resulting posterior distribution of component C after the Bayesian update. Note how the additional data works to reduce the uncertainty around the estimate. Figure 3 also shows how this additional testing is reflected as reduced uncertainty at the subsystem level D. A numerical summary of the Bayesian update is shown in Table 2.

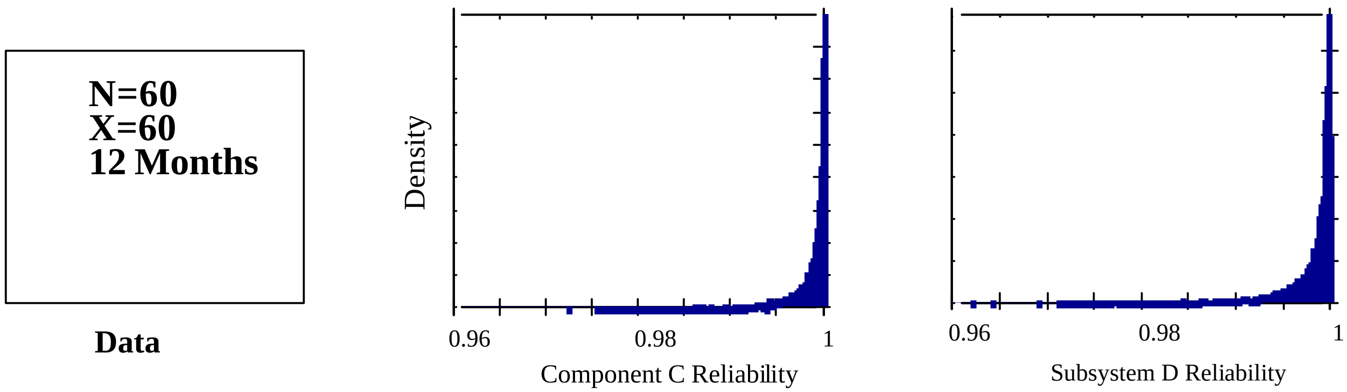

Figure 3. Reliability Posterior Distributions @ 12 Months

This methodology was used throughout the activity to provide estimates of reliability with uncertainty for all components, subsystems, and the system at various operating times. The median system reliability and lower 90 % confidence limit were also plotted against calendar time (as update events occurred) to track progress and demonstrate reliability growth as shown in Figure 4. The individual data points correspond to the initial reliability characterization R_0 and the events associated with the updates R_i. This plot captures the results of the design teams' early efforts to improve reliability, but the power of the approach is the roadmap developed which may be used by the team to organize their planning to achieve higher reliability.

Table 2. Prior and Posterior Reliability Distributions (Testing of Component C)

	Prior R0						Posterior R1					
	12 Month			100,000 Miles			12 Month			100,000 Miles		
Percentiles	5	50	95	5	50	95	5	50	95	5	50	95
Component A	0.9996	0.9999	1	0.9993	0.9999	1			Same			
Component B	0.9989	1	1	0.9986	0.9999	1			Same			
Component C	0.976	0.9989	0.9999	0.8829	0.9952	0.9997	0.9908	0.9992	0.9999	0.9599	0.9964	0.9997
Subsystem D	0.9723	0.9985	0.9998	0.8794	0.9944	0.9994	0.9887	0.9989	0.9998	0.957	0.9957	0.9994

8 SOME USEFUL APPROXIMATIONS

While the methodology described in Section 7 does not require $f(R_i)$ to conform to any particular distributional form or family, a useful approximation which sometimes may be helpful for planning purposes can be organized around the beta and binomial distributions, eq (3) and eq (4) respectively.

$$\text{Beta}\ (a, b) = \Gamma(a+b)/[\ \Gamma\ (a)\ \Gamma\ (b)]\ \ p^{\ a-1}\ (1-\ p)^{\ b-1} \tag{3}$$

$$\text{Binomial}\ (n, p)\ =\ \frac{n!}{x!\ (n-p)!}\ p^{\ x}\ (1-p)^{\ n\ -x} \tag{4}$$

The beta distribution is the conjugate prior distribution for the binomial parameter, p, (Martz et al. 1990) and

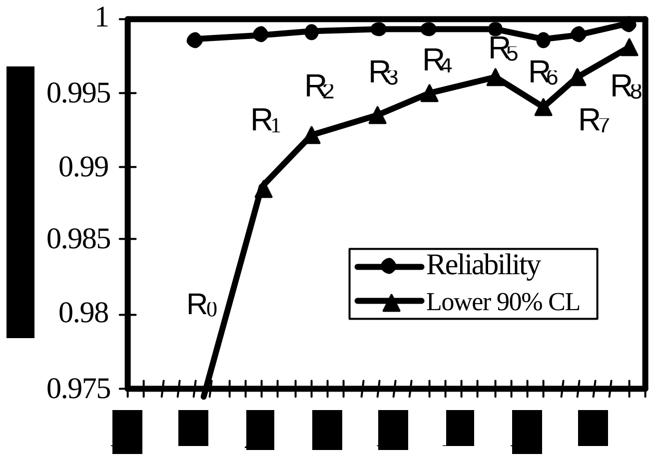

Figure 4. Reliability Growth

can in some cases be used to approximate the empirical distribution (resulting from the simulation) of the R_i. The beta is often well-suited for representing possible values for p because it ranges between 0 and 1, and in addition, it is an extremely flexible distribution with many possible shapes (e.g., symmetric, asymmetric, unimodal, uniform, U-shaped, or J-shaped). Its usefulness derives from the fact that the two parameters of the beta in eq (3) , a and b, are sometimes referred to as the *pseudo successes* and *pseudo failures*, respectively. This calls to mind the image of a *pseudo test*, where $a + b$ equals the number of pseudo tests.

A useful planning application involves situations where new test data is, or will be, of the form of x number of successes out of n number of trials. Such data is binomially (eq (4)) distributed. In a Bayesian reliability formulation, if a beta distribution with parameters a and b is considered to be the prior distribution for R_0, then the posterior distribution for R_1 (formed from a test of x successes in n trials) will also be a beta, with parameters $a + x$ and $b + n - x$. Thus, using the beta formulation may be useful in characterizing the possible value of additional tests. Because the posterior distribution and the prior distribution are both of the beta family, this process could be iterated indefinitely.

For example, the beta distribution shown in Figure 5 was fit to the prior reliability distribution for component C in Figure 2 (design portion only). In this case, a beta approximation yielded, a = 28.2 pseudo successes and b= 0.22 pseudo failures (a pseudo test of about 28 samples). New information, in the form of a 12 month test of 60 of these components resulting in zero failures was introduced, and a new predicted posterior beta reliability distribution was determined, also shown in Figure 5, using the methodology described above. Note that the beta parameters of this predicted posterior distribution are a = 88.2 and b = 0.22. This is obviously quite similar to the corresponding fitted posterior reliability distribution calculated empirically for component C and also shown in Figure 5. It is also possible to streamline the calculations of the posterior distribution of subsystem D by using this beta estimate. The power of this approximation, however, lies in simply noting the potential impact of this test (visually or through the beta parameters) and allowing the engineering community to judge the usefulness of this test before it is run.

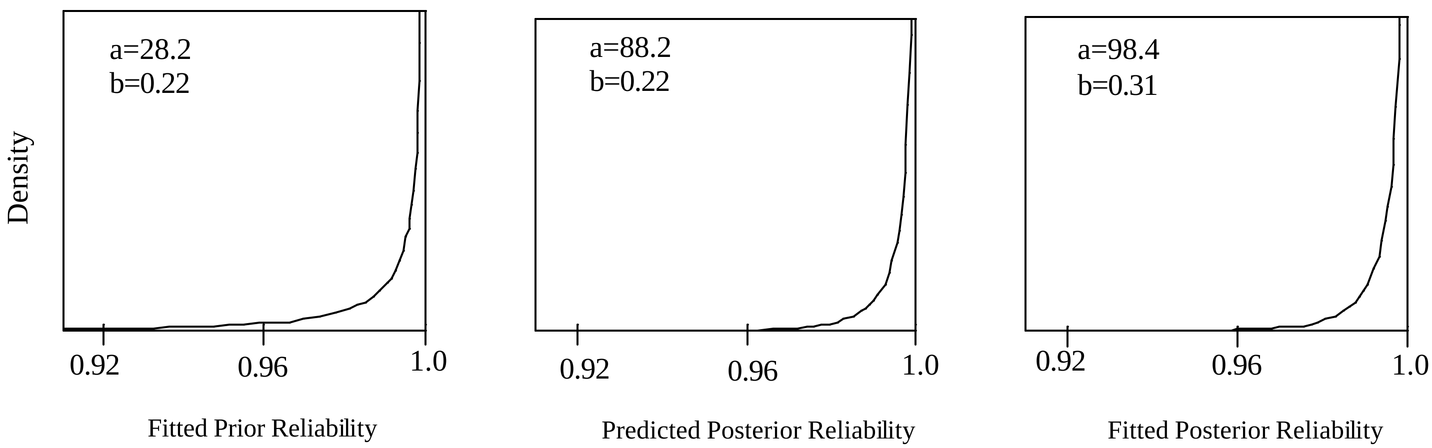

Figure 5. Component C Beta Distributions (Design Failure)

Another useful approximation which sometimes may be helpful for planning purposes can be organized around the gamma and exponential distributions, eq (5) and eq (6) respectively,

$$\text{Gamma}\,(a,\,h) \;=\; \frac{h^{a}\;\,l^{\,a-1}\,\exp\,(-h\,l)}{\int x^{\,a-1}\,\exp\,(-x)\,dx} \tag{5}$$

$$\text{Exponential}\,(t, l) \;=\; \lambda\;\exp\,(-\;l\;t) \tag{6}$$

or the gamma and Weibull distribution eq (7) in what is referred to as *transformed time.*

$$\text{Weibull}\,(t, l, b) \;=\; l\,b\,(\,t\,)^{b-1}\;\exp\,(-\;l\,(\,t\,)^{b}) \tag{7}$$

The gamma distribution is the conjugate prior distribution for the exponential parameter, l, and can in some cases be used to approximate the empirical distribution (resulting from the simulation) of the R. The gamma is often well-suited for representing possible values for l because it ranges between 0 and infinity.

Suppose the test planning situation involves test data that is, or will be, of the form of the number of successes in a test run for a specified length of time. Such data is distributed according to the Weibull model eq (7) where l is the failure rate as specified by the data and b is the decay of that rate. Note that this parameterization of the Weibull reduces to the exponential distribution eq (6) when b =1. Note also that for a <u>constant</u> value of b, l in the Weibull expression eq (7) is equivalent to the l in the exponential expression eq (6) for *transformed time*, t^{b}. In the Bayesian reliability formulation with $b = 1$ (exponential), if a gamma distribution eq (5) with parameters a and h is considered to be the prior distribution for l, then the posterior distribution for l will also be a gamma, with parameters $a + s$ and $h + t$, where s failures are observed during t total time on test ($t = \Sigma\, t_i$ and t_i is the time on test for the i th test unit). The usefulness of this arrangement derives from the fact that the two parameters of the gamma in eq (5) , a and h, are sometimes referred to as the *pseudo failures* and *pseudo total test time,* respectively. This calls to mind the image of a *pseudo test*, where a failures are experienced during h amount of total test time.

Analogous results hold for the Weibull when b is <u>constant</u> and known. Such a failure model is equivalent to an exponential with a transformed time variable, or with t replaced by t^{b}. In this Bayesian case, if a gamma distribution with parameters a and h is considered to be the prior distribution for l, then the posterior distribution for l will also be a gamma, with parameters $a + s$ and $h + t$ where s failures are observed during t total <u>transformed</u> time on test ($t = \Sigma\, (t_i)^{b}$, and t_i is the time on test for the i th test unit). The usefulness of this arrangement again derives from the fact that the two parameters of the gamma, a and h, are sometimes referred to as the *pseudo failures* and *pseudo total <u>transformed</u> test time,* respectively. This again calls to mind the image of a prior *pseudo test*, which may be useful in characterizing the possible value of additional tests. Because the posterior distribution and the prior distribution are both of the gamma family, this process could also be iterated indefinitely. Various limitations of these examples are discussed in Kerscher et al. (1998).

Characterizing with large uncertainty the initial reliability of a new product under development, and then working to reduce that uncertainty, has been found to be a culturally acceptable way to address the reliability issue. These examples illustrate cases where new test information or data are introduced to update a reliability, R_i, to the form R_{i+1}. The continuous monitoring of these reliability snapshots, R_i and $f(R_i)$, is possible as new information or changes become available. Not all changes may be beneficial, as reliability can decrease and / or the uncertainty increase at any given change step, i. However, by judiciously planning new tests, analyses or changes for the purposes of reducing uncertainty and / or improving reliability, the overall trend will indicate such desired results (reliability growth). This overall methodology may prove useful in characterizing the reliability of a new product in its concept stage, updating and reporting on that reliability during the development stage, and facilitating the planning of appropriate future activities which, when accomplished, will drive reliability higher. If application of this methodology allows a project team to successfully include the reliability issue in its day-to-day activities involving performance, cost, and timeliness, it will prove to be a powerful tool in the development of a high reliability product.

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