UNCERTAINTY AND RELIABILITY

OUTLINE

- Sources of Uncertainty
- Background information
  1. Probability basics
  2. Reliability/Performance Models
- Different sources of information

SOURCES OF UNCERTAINTY

- “Statistical” Variability

  1. sampling variability
  2. measurement error

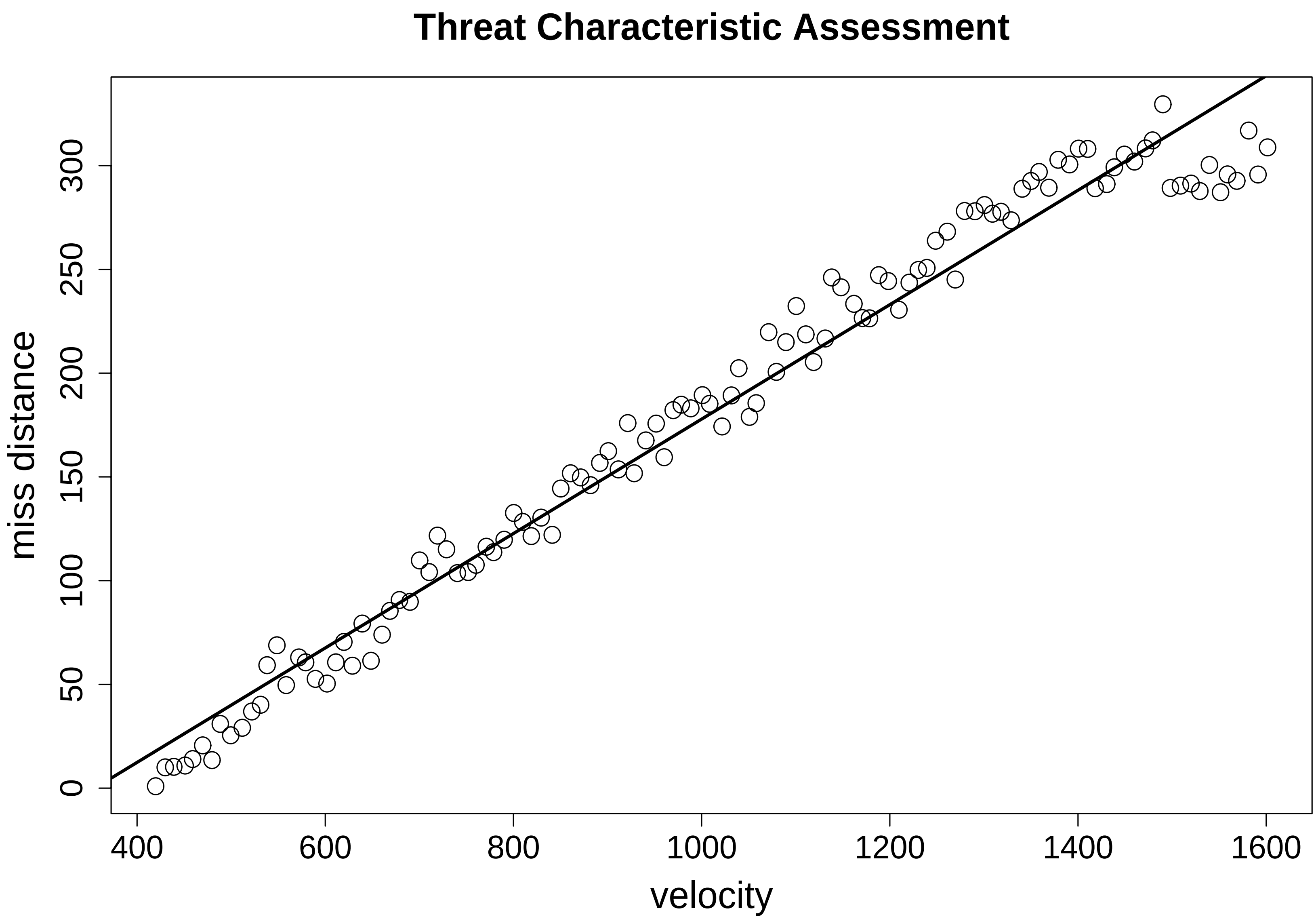
- Data
- Bias
- Model

SOURCES OF UNCERTAINTY

Example

- Want to understand the effect of threat characteristics on missile performance
- Response variable: miss distance
- Explanatory variable: velocity of the threat

SOURCES OF UNCERTAINTY



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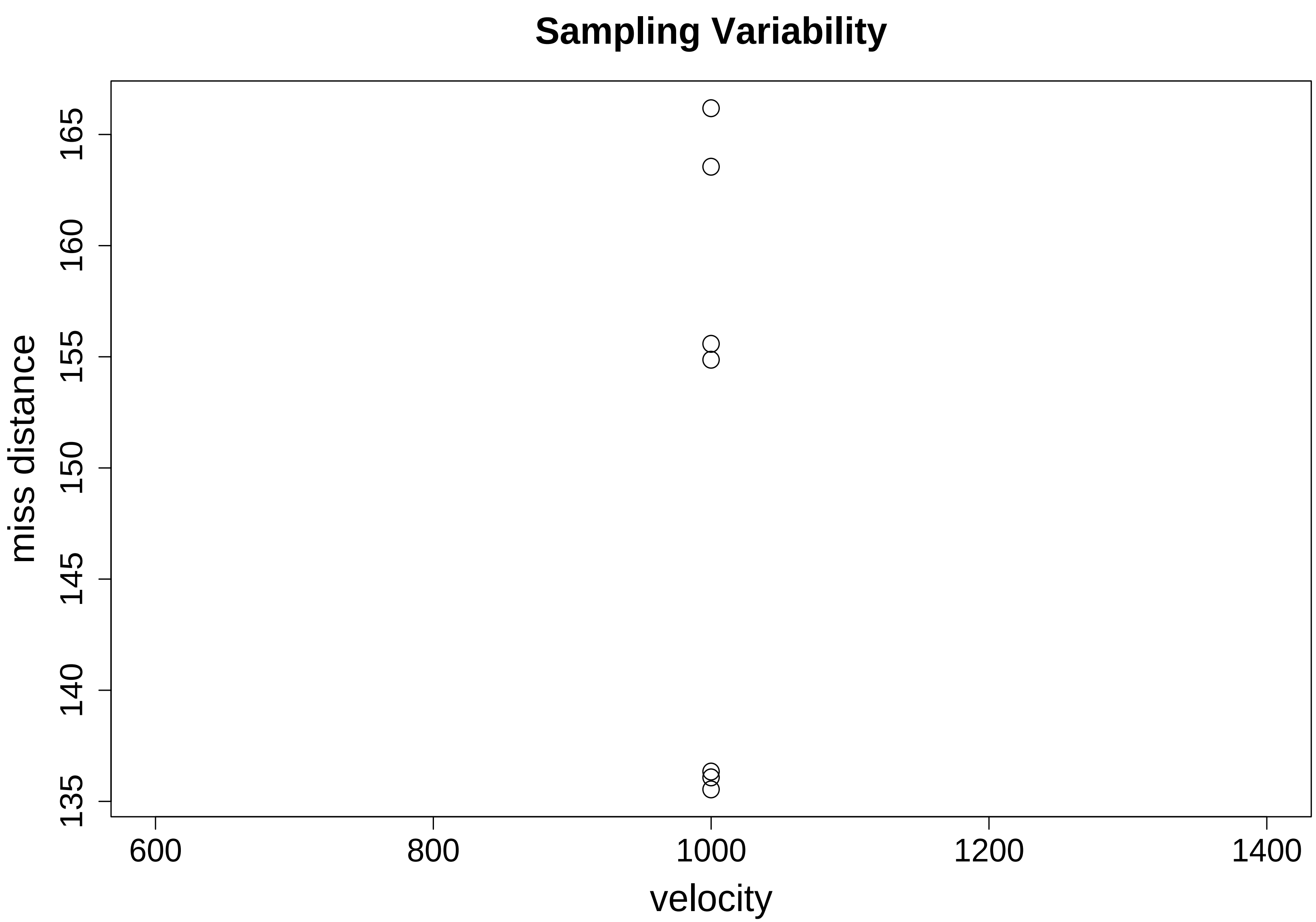

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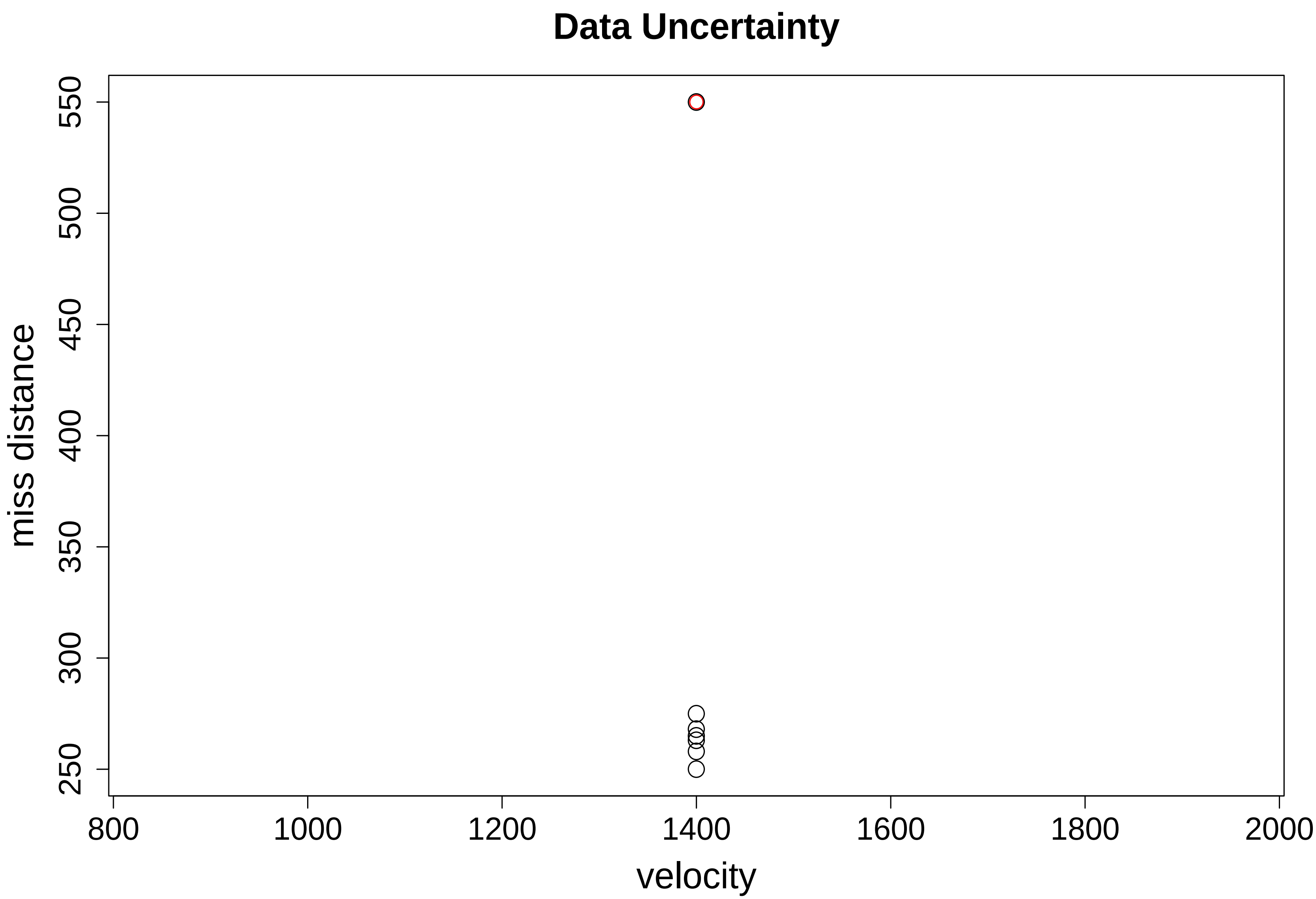


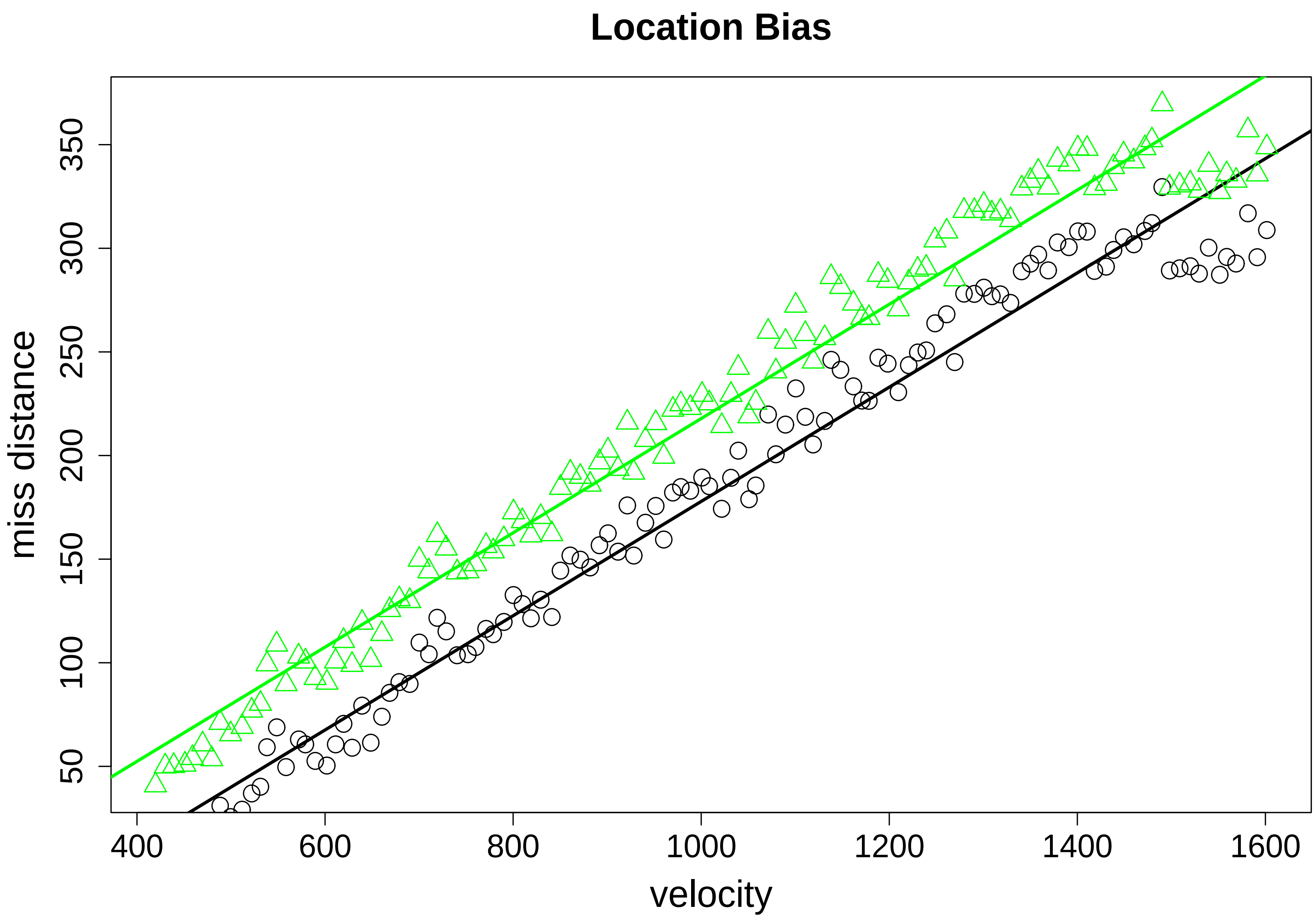
SOURCES OF UNCERTAINTY

Computer Models May Induce Bias

- Physical experiments: everything's in there, we just don't know what's in there
- Computer experiments: **not** everything's in there, but we know everything that's in there.
- two kinds of bias:

  1. location bias
  2. scale bias

SOURCES OF UNCERTAINTY



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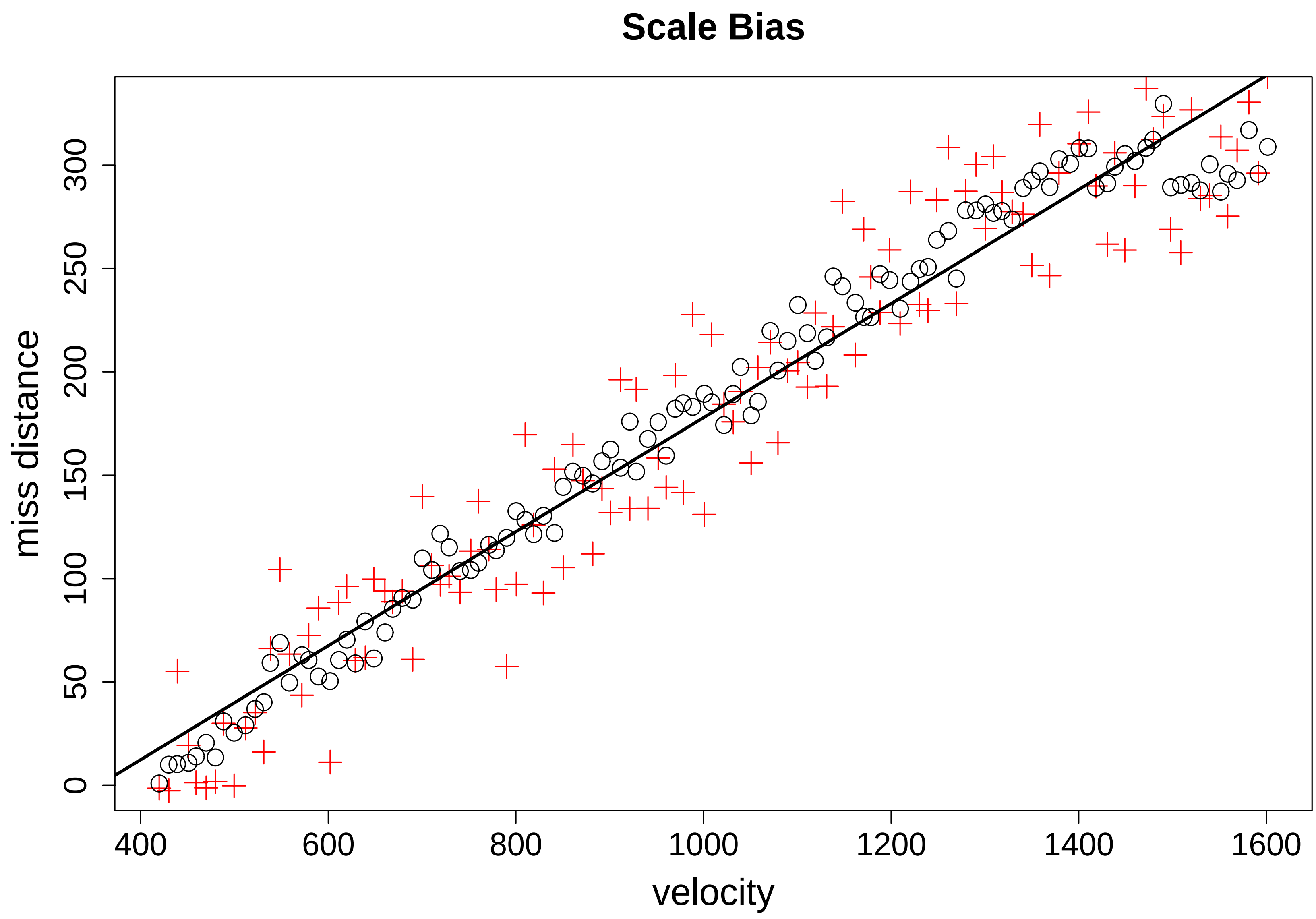

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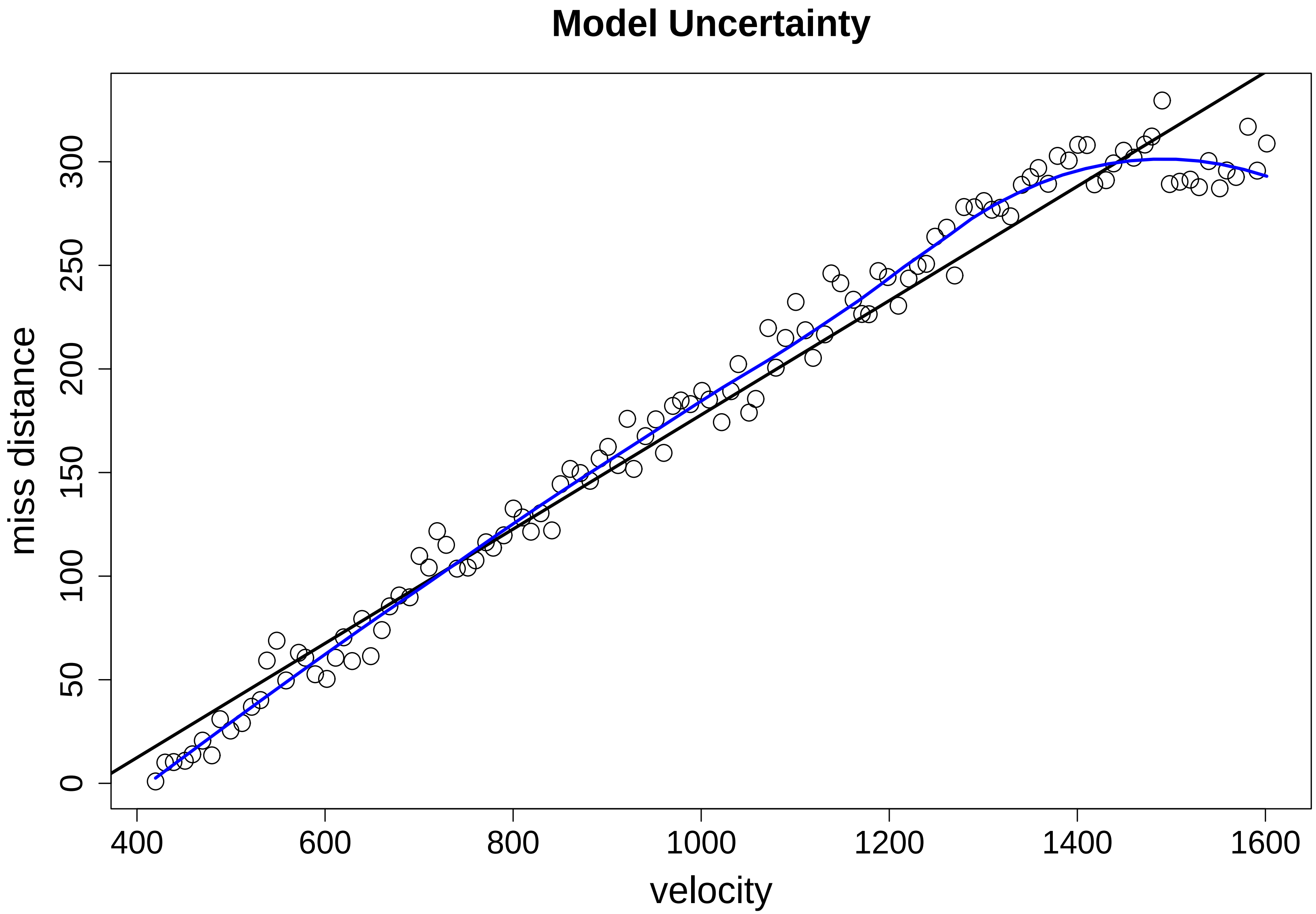


DISCUSSION

- Difficulties

  1. Statistical – need data (sometimes *a lot*)
  2. Data – exactly what constitutes “strange”?
  3. Bias – identifiability
  4. Model – by far the hardest to assess

- WIP

CONDITIONAL PROBABILITY

- Independence: $Pr(A \cap B) = Pr(A) \times Pr(B)$
- Conditional Probability

$$Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)}$$

Note: if A and B are independent, then $Pr(A|B) = Pr(A)$.

RELIABILITY MODELS

- What is reliability?

$$R(t) = Pr(T \geq t) = \int_0^t f(x)dx$$

where $f(x)$ is the distribution of failure times.

- Easy textbook definition, hard in practice.

1. What is a failure?
2. Is there a mapping between performance and reliability?

RELIABILITY MODELS

slide indicating problems defining reliability

reliability vs. performance

Hazard Rate

- Define *hazard rate* as:

$$h(t) = \frac{f(t)}{R(t)}$$

- So what?

1. Instantaneous failure rate
2. Great for model identification

Reliability Basics

- T represents our *random variable* of interest.
- Often T is the time until failure or failure time.
- Probability distribution of failure times ($f(t)$).
- t represents the *realization* of the random variable.

RELIABILITY – BINOMIAL

- Model for success/failure data
- n trials, X successes
- $p = Pr(\text{success})$ is the same for all trials
- Trials are independent
-

$$Pr(X = c) = \binom{n}{c} p^c (1 - p)^{n-c}$$

RELIABILITY – EXPONENTIAL

- Reliability

$$R(t|\lambda) = \exp^{-\lambda t}, t > 0$$

- Hazard Rate

$$h(t|\lambda) = \lambda$$

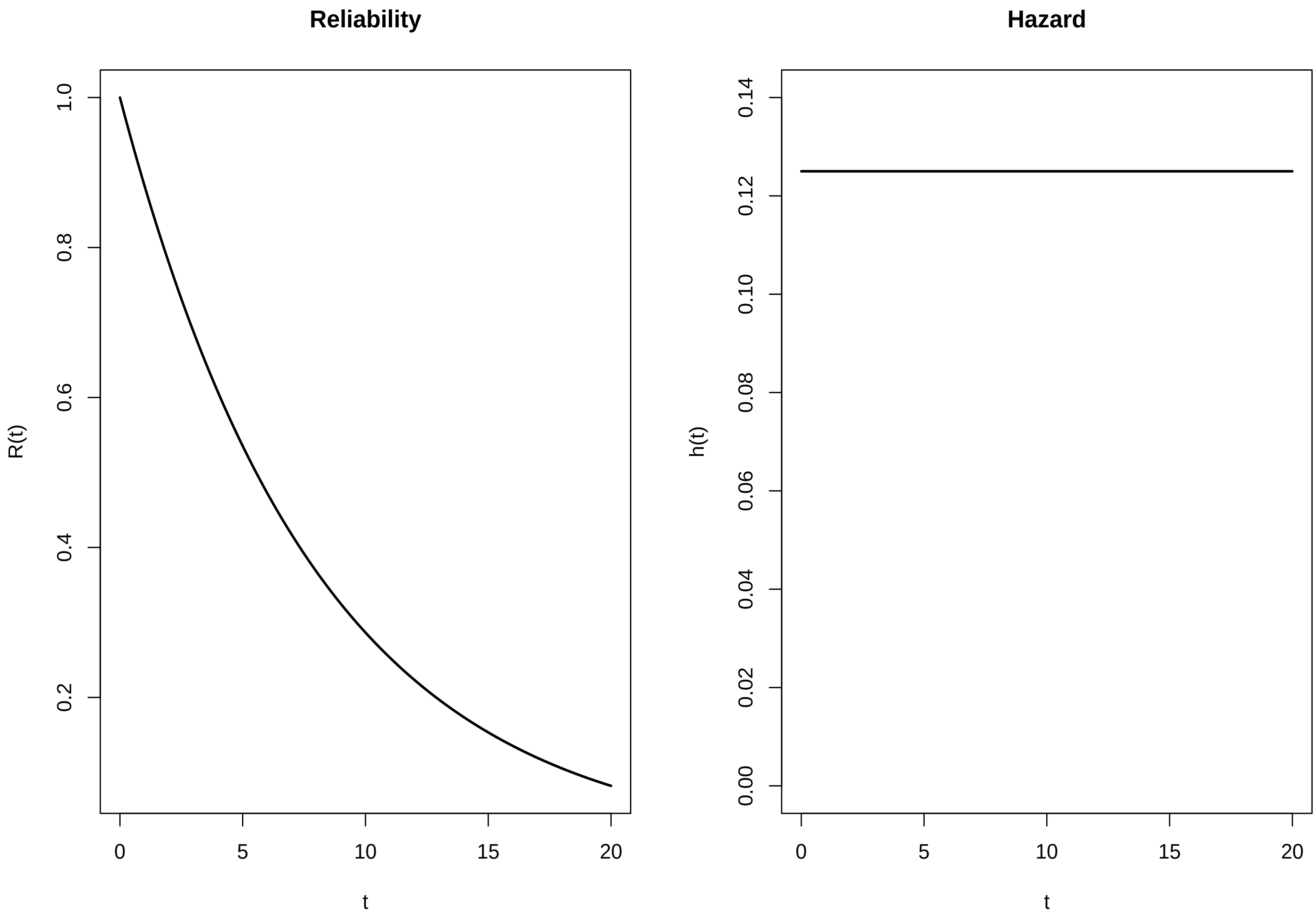
- Mean Time to Failure (MTTF)

$$E(T) = \frac{1}{\lambda}$$

- Variance of TTF

$$V(T) = \frac{1}{\lambda^2}$$

RELIABILITY – EXPONENTIAL



RELIABILITY – WEIBULL

- Reliability

$$R(t|\alpha,\theta) = \exp\left[-\left(\frac{t-\theta}{\alpha}\right)\right], t > \theta$$

- Hazard Rate

$$h(t|\alpha,\theta) = \frac{\beta}{\alpha}\left(\frac{t-\theta}{\alpha}\right)^{\beta-1}, t > \theta$$

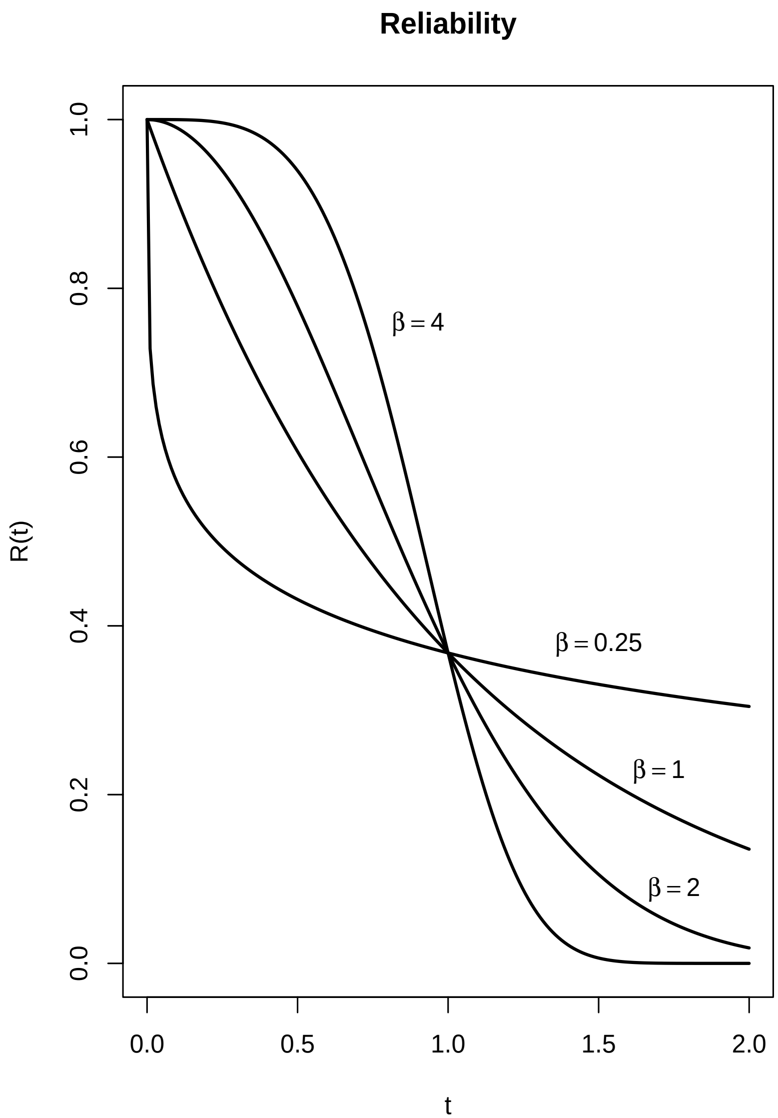
- Mean Time to Failure (MTTF)

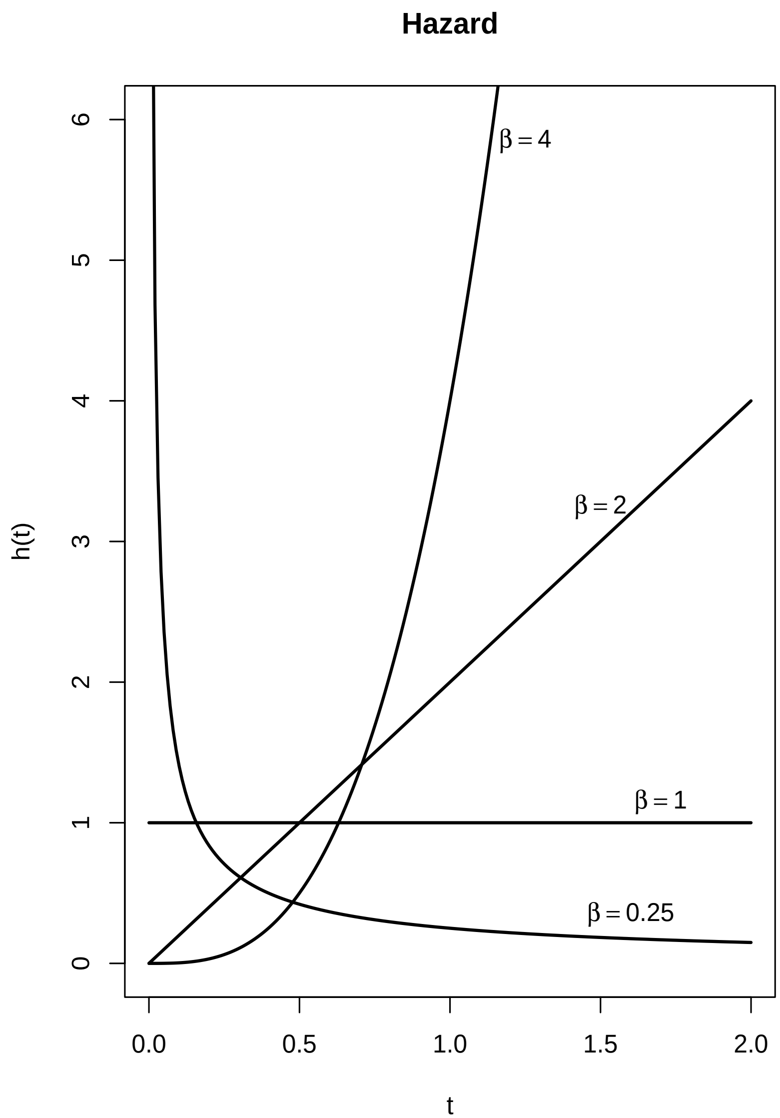
$$E(T) = \theta + \alpha\Gamma\left(\frac{\beta+1}{\beta}\right)$$

- Variance of TTF

$$V(T) = \alpha^2\left[\Gamma\left(\frac{\beta+2}{\beta}\right) - \Gamma^2\left(\frac{\beta+1}{\beta}\right)\right]$$

RELIABILITY – WEIBULL





RELIABILITY – LOGNORMAL

- Reliability: if $\log(Y) \sim Normal$ then $Y \sim LN$

$$R(t|\xi,\sigma) = 1 - \Phi\left(\frac{\log t - \xi}{\sigma}\right)$$

- Hazard Rate

$$h(t|\xi,\sigma) = \frac{\phi\left(\frac{\log t - \xi}{\sigma}\right)}{\sigma t - \sigma t \Phi\left(\frac{\log t - \xi}{\sigma}\right)}$$

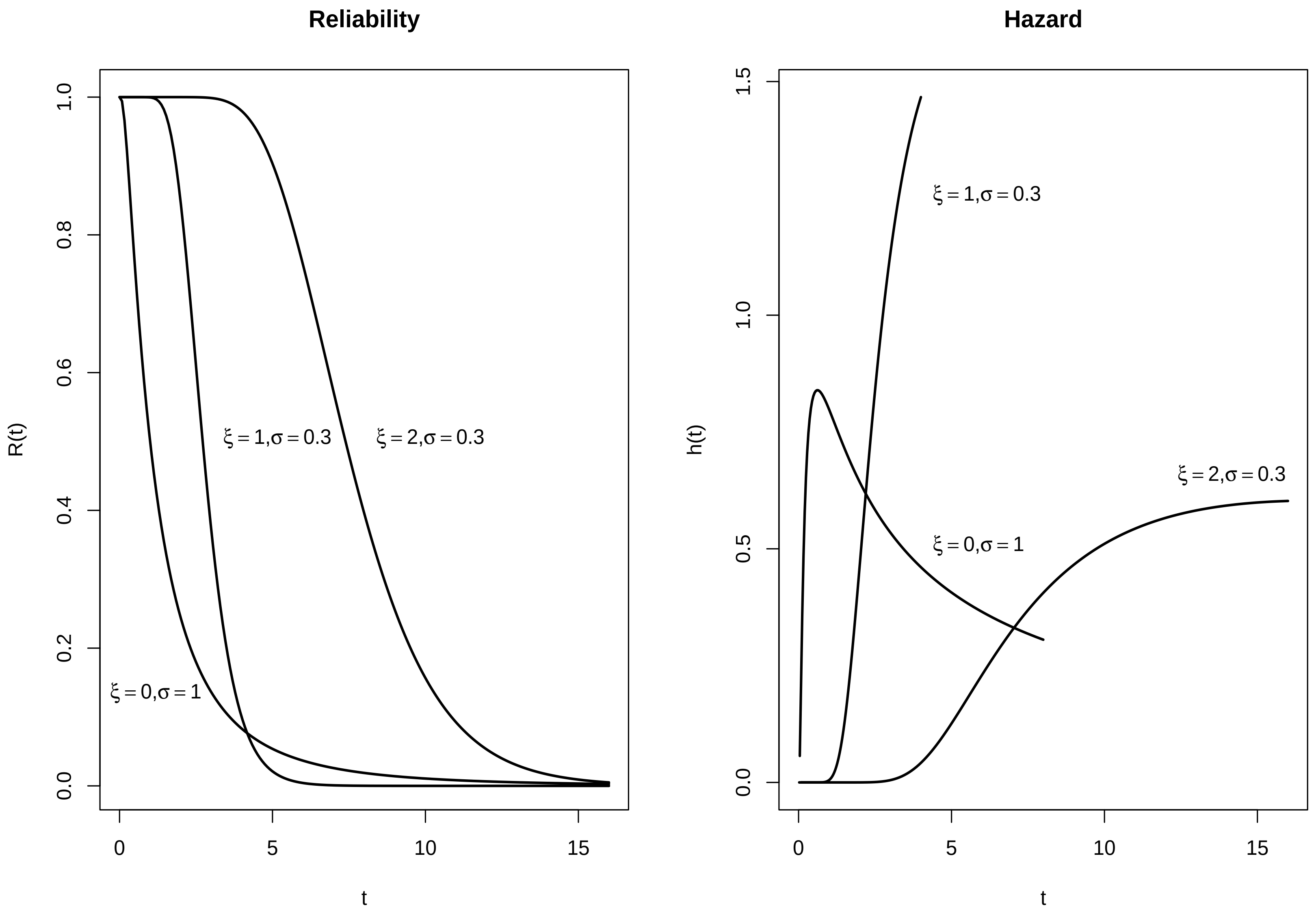
- Mean Time to Failure (MTTF)

$$E(T) = \exp\left(\xi + \sigma^2/2\right)$$

- Variance of TTF

$$V(T) = (e^{2\xi+\sigma^2})(e^{\sigma^2} - 1)$$

RELIABILITY – LOGNORMAL



RELIABILITY – GAMMA

- Reliability:

$$R(t|\alpha,\beta) = \frac{\Gamma(\alpha) - \Gamma(\alpha, t\beta)}{\Gamma(\alpha)}$$

- Hazard Rate

$$h(t|\alpha,\beta) = \beta^{\alpha} \frac{t^{\alpha-1}\exp(-t\beta)}{[\Gamma(\alpha) - \Gamma(\alpha, t\beta)]}$$

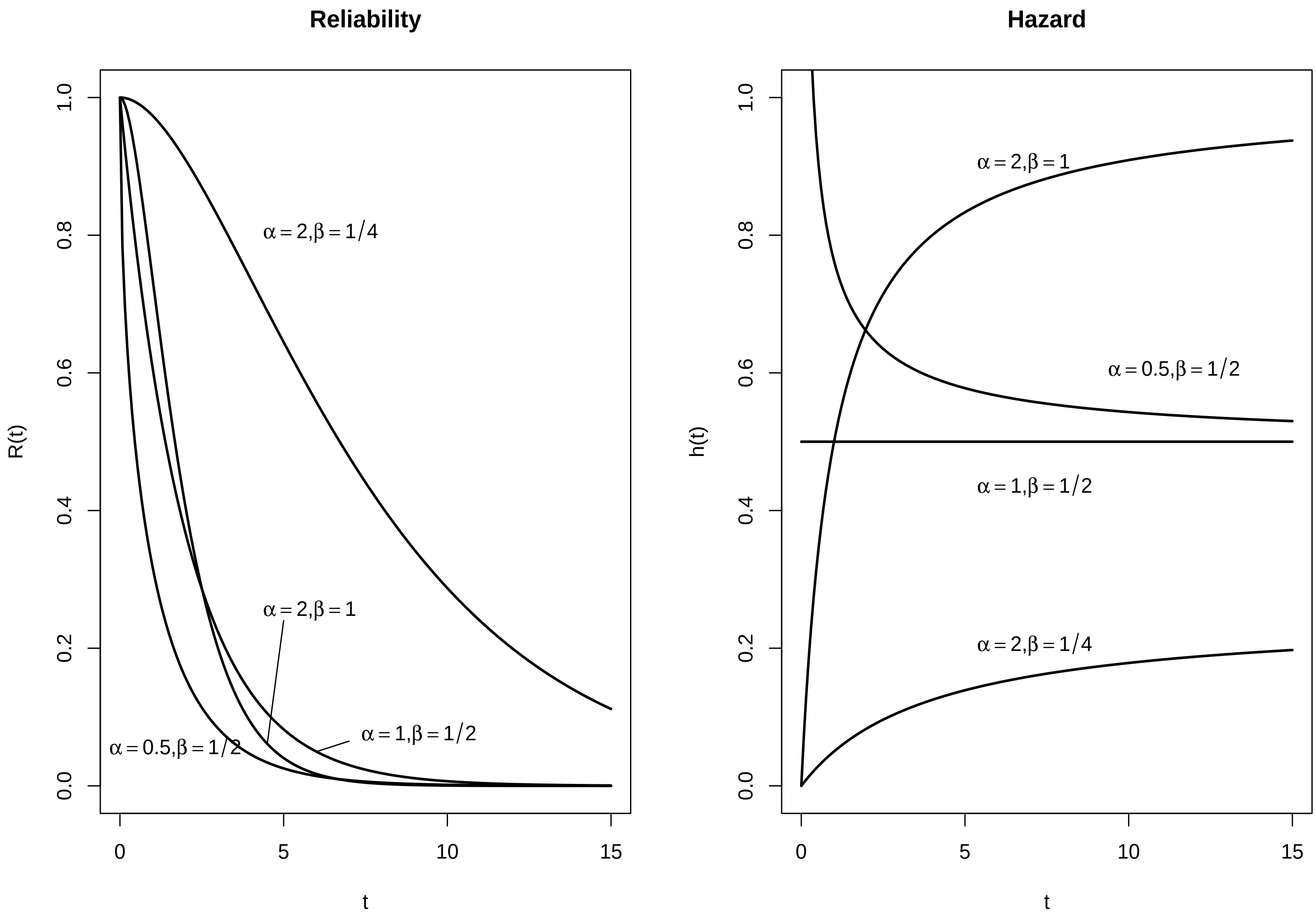
- Mean Time to Failure (MTTF)

$$E(T) = \frac{\alpha}{\beta}$$

- Variance of TTF

$$V(T) = \frac{\alpha}{\beta^2}$$

RELIABILITY – GAMMA



Performance Basics

- Y represents our *random variable* of interest.
- Represents the level of performance and its uncertainty.
- Probability distribution of failure times ($g(y)$).
- y represents the *realization* of the random variable.

PERFORMANCE – NORMAL

- Performance Distribution:

$$g(y|\mu,\sigma) = (2\pi)^{-\frac{1}{2}} \exp\left\{-\frac{(y-\mu)^2}{2\sigma^2}\right\}$$

- Mean performance

$$E(Y) = \mu$$

- Variance of performance

$$V(Y) = \sigma^2$$

- Sometimes called “Gaussian”.

PERFORMANCE – BETA

- Performance: [support on (0,1)]

$$g(y|n_o, x_o) = \frac{\Gamma(n_o)}{\Gamma(x_o)\Gamma(n_o - x_o)} y^{x_o - 1}(1-y)^{n_o - x_o - 1}$$

- Mean performance

$$E(Y) = \frac{x_o}{n_o}$$

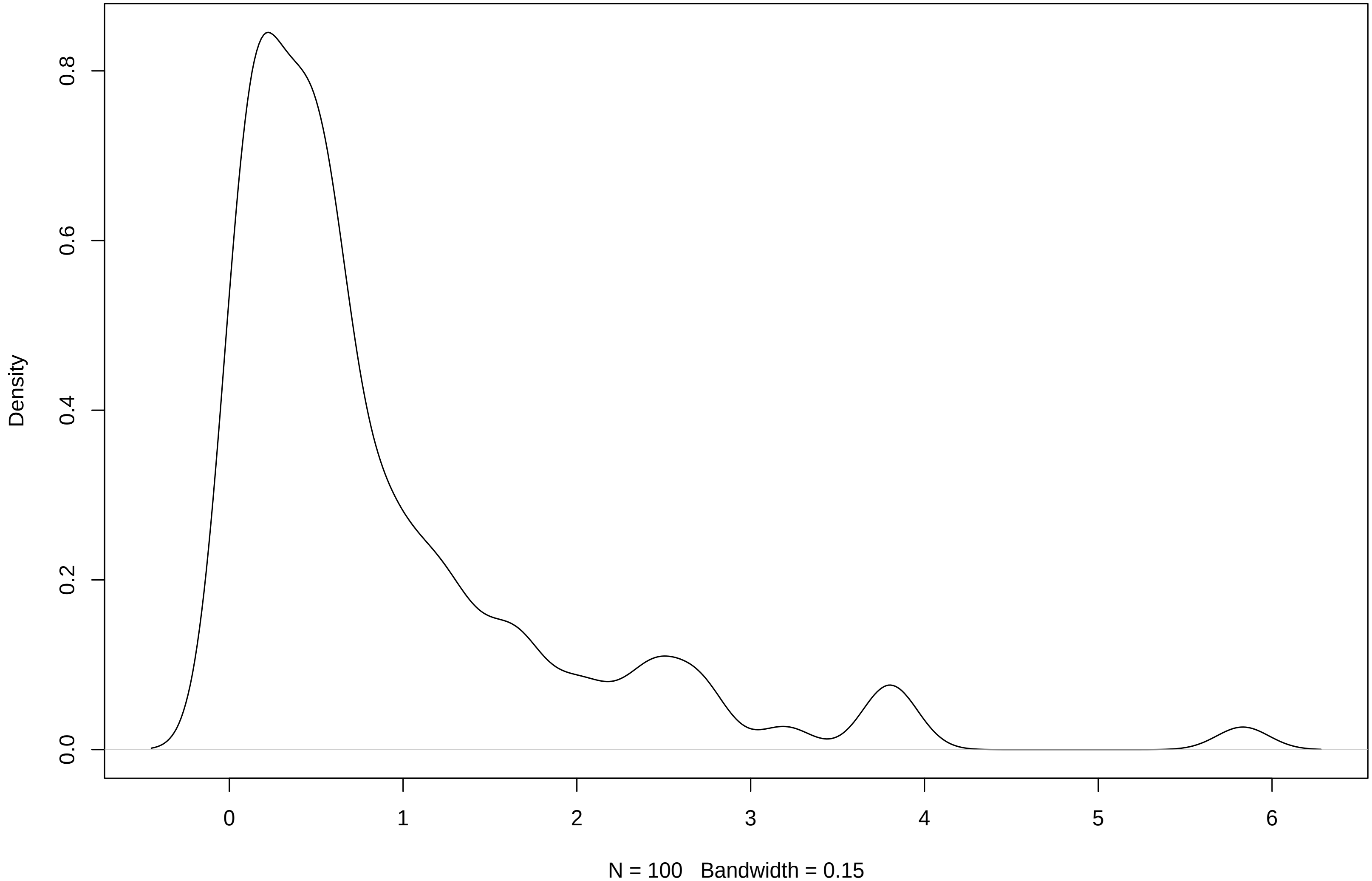
- Variance of performance

$$V(Y) = \frac{x_o(n_o - x_o)}{n_0^2(n_o + 1)}$$

EMPIRICAL DISTRIBUTIONS

- Probability distributions must integrate to one
- Why force them into one of the above forms?
  (Besides making it easier to teach the course?)
- Bumps and tails
- Kernel density estimators

EMPIRICAL DISTRIBUTIONS



LINEAR MODELS

- Recall: We wanted to understand the miss distance of our Really Deadly Missile System as a function of the threat characteristic: Threat velocity.

- Are there other threat characteristics of interest?

  1. Velocity (V)
  2. Radar cross section (C)
  3. Penetration aids (A)
  4. Pitch (P)

LINEAR MODELS

- When we only have one threat characteristic (velocity), we have

$$Y_i \sim N(\beta_0 + \beta_1 V_i, \sigma^2)$$

- Now, how to compactly represent the situation where we are interested in all 4 threat characteristics?

$$Y_i \quad \sim \quad N(\beta_0 + \beta_1 V_i + \cdots + \beta_4 P_i, \sigma^2)$$

- Not very compact (20 explanatory vars)

LINEAR MODELS

- Better:

$$\underline{\mathbf{Y}} \sim N(\mathbf{X}\underline{\beta}, \sigma^2 \mathbf{I}),$$

where $\mathbf{X}$ contains all 4 explanatory variables and

$$\mathbf{I} = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & \cdots & 1 \end{pmatrix}$$

DISCUSSION

- We've just discussed statistical models for "data".
- What types of things constitute "data"?

1. Complex computer codes
2. Physical experimental data
3. Expert judgement
4. Other

DISCUSSION

- While most of the discussion for the class will focus on simple parametric models, keep in mind that

  1. empirical distributions and non-parametric models are often useful, and
  2. more complicated models are used frequently, and come up naturally as different data sources are combined.