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Title: EXAMPLE OF USING FUZZY CONTROL SYSTEM METHODS IN STATISTICS

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Example of Using Fuzzy Control System Methods in Statistics

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Introduction

Probabilists and fuzzy logic enthusiasts have argued about which philosophy is best. Both tools have their place in the world of problem solving. In many cases, fuzzy logic is selected by the practitioner over a probabilistic approach because the probability literature is perceived as being too theoretical or impractical. Alternatively, some problems are solved using probability because fuzzy techniques are perceived as being too empirical. Sometimes both tools can be used together, synergistically [1,2], as the engineering and statistical authors are doing for the mutual benefit of both disciplines.

Fuzzy control system techniques are used to synthesize systems for enhanced control of processes. These techniques are especially useful for highly nonlinear systems or systems whose mathematical models are either inaccurate or unavailable. The control system maps observed plant output parameter values into required control actions, or plant inputs. The rules and functions for controlling plant operations can then be burned onto a chip.

Application of Fuzzy Methods for Uncertainty Distributions

Fuzzy methods can be used in a statistical context [2], and in particular, for the development of uncertainty distributions in reliability applications where test data are sparse and reliance is heavy on human judgment for predicting system performance [1]. In such an application, the plant output parameters used by the control system become component condition, and the control actions become the predicted component response or performance. In essence, we are mapping the component conditions into system performance.

The component conditions are transformed into degrees of membership in fuzzy component condition sets via membership functions (Figure 1). *If-then* rules transform these degrees of membership into weights associated with the corresponding performance level sets. The performance level sets are also characterized by membership functions (Figure 2). The set of possible responses, the performance level set, is characterized by a weighted combination of the corresponding membership functions. Fuzzy control adjusts performance via a *defuzzification* process, such as selecting the centroid of the combined performance membership function. However, we are interested in the entire combined function, representing the uncertainty in performance.

Fuzzy rule-based methods permit experts to assess parameters affecting performance of components, subsystems, and the overall system in semantic terms more familiar to them, e.g., “high” or “good.” Formal, structured techniques for eliciting performance parameters from the experts can be found in [3]. We have adapted rule-based methods to systems that have a high cost for obtaining more precise information and where the mathematical relationship between the condition of the components and their performance is not well understood (e.g., some sets of operating conditions have not been experienced but are anticipated as part of the system’s aging). The fuzzy rule-based methods allow the experts’ understanding of the underlying processes to be represented with precision [1], even when that understanding includes a high degree of uncertainty.

An Example

Consider a system with one component that can influence performance of the system. The component is subject to wear, potentially degrading performance. For a given condition level, performance degradation will be variable. Figure 1 shows membership functions for three component condition sets, {A="none", B="moderate", C="severe wear"}. Figure 2 shows membership functions for three performance level sets, that might correspond to responses, {a="acceptable", b="marginal", c="poor"}.

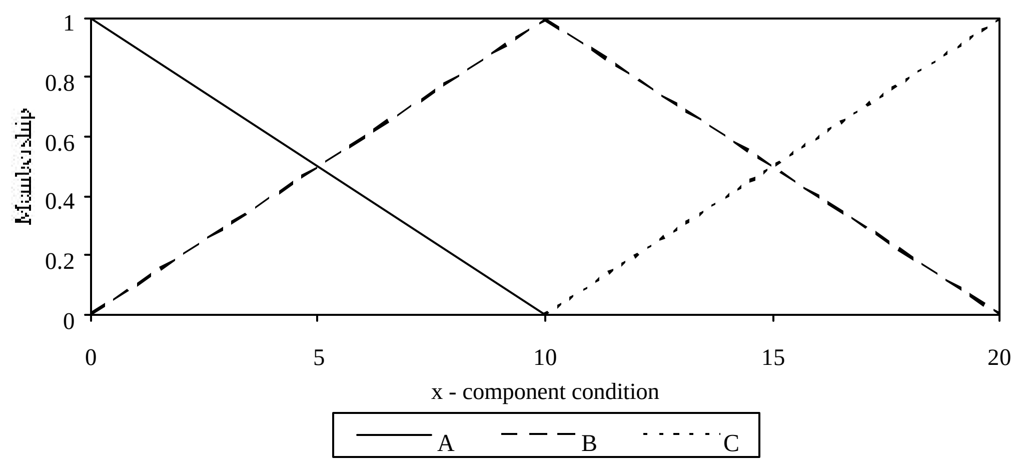

Figure 1. Component condition sets for 3 membership functions

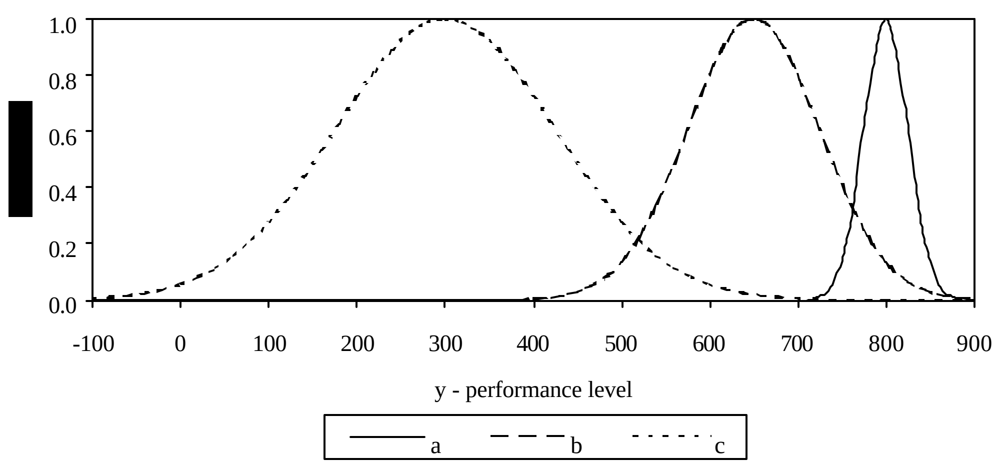

Figure 2. Performance level sets for 3 membership functions

Three *if-then* rules define the condition/performance relationship:

if condition is A, then performance is a;
if condition is B, then performance is b; and
if condition is C, then performance is c.

If component condition is x = 4.0, then x has membership of 0.6 in A and 0.4 in B. Using the rules, the defined component condition membership values are mapped to performance level weights. Following fuzzy system methods, the membership functions for performance level sets a, N(800,25)[1], and b, N(650,75), are combined based on the weights 0.6 and 0.4. This combined membership function can be used to form the basis of an uncertainty distribution for characterizing performance for a given condition level.

A somewhat equivalent probabilistic approach involving mixtures of distributions can be developed with proper construction of the membership functions [4]. In addition, linear combinations of random variables provide an alternative combination method [2], when mixtures produce multi-modality results—which can be undesirable from a physical interpretation standpoint.

Departing from standard fuzzy systems methods, we normalize the combined performance membership function so that it integrates to 1.0. The resulting function, f(y|x), is the uncertainty distribution for performance, y, corresponding to the situation where component condition is equal to x. Figure 3 is the cumulative distribution function of the uncertainty distribution, F(y|x). If performance must exceed some threshold, T, in order for the system to operate successfully, the reliability of the system for the situation where component condition is equal to x can be expressed as R(x) = 1- F(T|x). As illustrated in Figure 3, a threshold of T = 550 corresponds to a reliability of R(4.0) = 0.925.

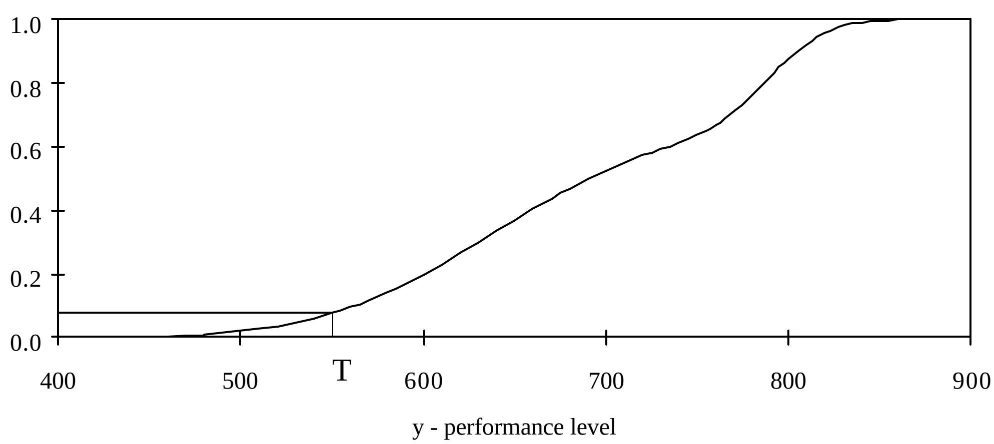

Figure 3. Uncertainty CDF for the condition x=4.0

Suppose that the uncertainty in wear, x, is characterized by some distribution, G(x). The results of repeatedly sampling x from G(x) and calculating F(y|x) produces an “envelope” of cumulative distribution functions. This “envelope” represents the uncertainty in the degradation probability that is due to uncertainty in the level of wear. The approximate distribution of R(x) can be obtained from such a numerical simulation.

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[1] The notation N(mean, standard deviation) for the performance level functions represents normal distributions without the scale factor so that they range from 0 to 1.

Summary

This example illustrates the case where the relationship between wear and performance is not well understood, and where some wear ranges are better understood than others. Experts find the semantic sets and rules useful in capturing their knowledge and uncertainties about conditions and system performance. We have successfully used the modified membership functions for more complex systems [1] and continue studying the effects of different methods for combining membership functions [2].

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4. Laviolette M., Seaman J. Jr, Barrett J., and Woodall W. "A Probabilistic and Statistical View of Fuzzy Methods," *Technometrics* 1995; **37**: 249-281.