Army Conference on Applied Statistics
Santa Fe, NM
October 25, 2001

The Role of Expert Knowledge in Uncertainty Quantification

(Are We Adding More Uncertainty or More Understanding?)

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Expert Knowledge

Expert Knowledge: what is known by qualified individuals, responding to complex, difficult (technical) questions, obtained through formal expert elicitation.

•A snapshot of the expert's state of knowledge at the time.

•Expressed in qualitative and quantitative form.

Expert Knowledge =
Expertise + Expert Judgment

Structure *(Expertise)*

- Define the problem
- Organize and represent the problem solving knowledge, the information flow
- Identify the relevant data and information (e.g., models, experimental results, numerical methods. . .)
- Identify **uncertainties** and determine how these are to be represented

Contents *(Judgment)*

- Provide quantitative and qualitative estimates and uncertainties, and the heuristics, assumptions and information used to arrive at answers to technical questions.

Uses of Expertise & Judgment

Expertise:

- Decision about what variables enter into a statistical analysis
- Decision about which data sets to include in an analysis
- Assumptions used in selecting a model or method
- Decision concerning which forms of uncertainty are appropriate to use (e.g., probability distributions)
- Description of experts' thinking and information sources in arriving at any of the above responses

Expert Judgment:

- Estimation of an occurrence of an event
- Estimation of the uncertainty of parameter
- Prediction of the performance of some product or process

Uncertainty Quantification

Broad Definition — the process of characterizing, estimating, propagating, and analyzing various kinds of uncertainty (including variability) for a complex decision problem.

For complex computer and physical models — focuses upon measurement, computational, parameter (including sensitivities of outputs to input values), and modeling uncertainties leading to verification and validation.

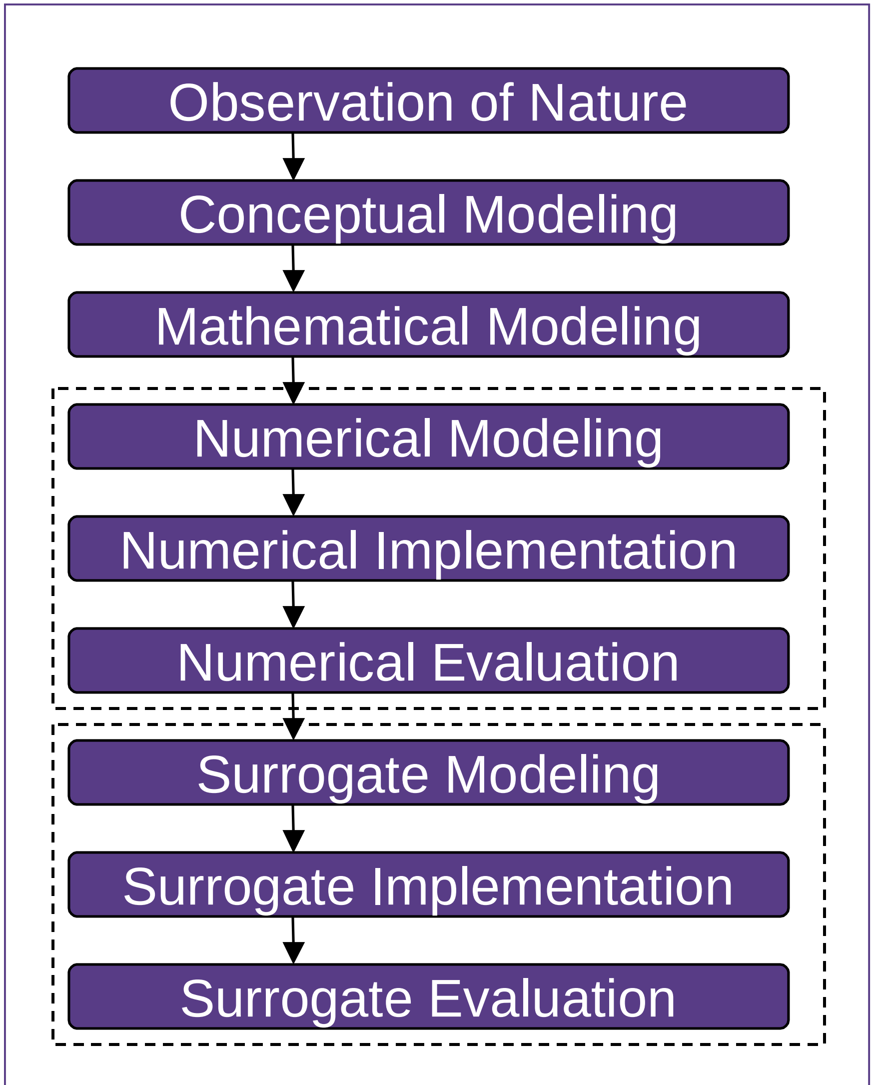
Two Categories of Uncertainty

- **Aleatory** —
  Inherent variation,
  Random,
  Irreducible
  (Includes variability)

- **Error** –
  numerical,
  discretization,
  mistakes

- **Epistemic** —
  Lack of knowledge,
  Reducible

The Modeling Process with Uncertainties



Sources of uncertainty

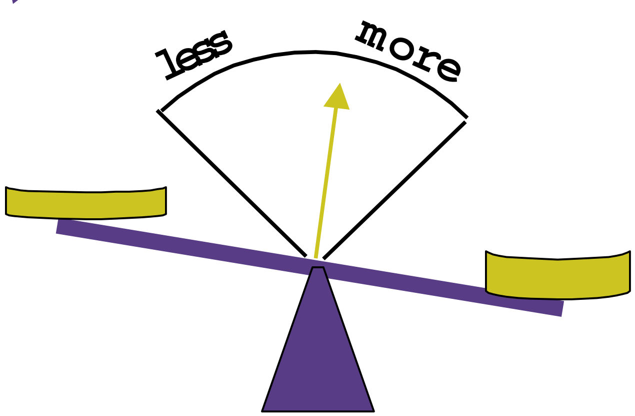
- **Measurements**
  - Noise
  - Resolution
  - Processing
- **Mathematical models**
  - Equations
  - Boundary conditions
  - Initial conditions
  - Inputs
- **Numerical models**
  - Weak formulations
  - Discretizations (mesh, time step)
  - Approximate solution algorithms
  - Truncation and roundoff
- **Surrogate models (statistical)**
  - Approximation error
  - Interpolation error
  - Extrapolation error
- **Model parameters**
- **Scenarios**

Additional Uncertainty: “Human In The Loop”

Sources of uncertainty

– **Measurements**
– **Mathematical models**
– **Numerical models**
– **Surrogate models (statistical)**
– **Model parameters**
– **Scenarios**

The expert is making decisions about all of these choices and inducing uncertainties in the process.

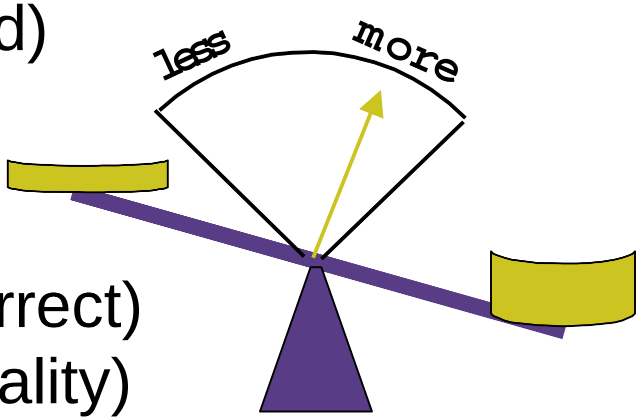


Cognitive and Motivational Biases Contribute

Bias: *A skewing from a standard or reference point. Can degrade the quality of the information and contribute to uncertainty.*

Cognitive biases:

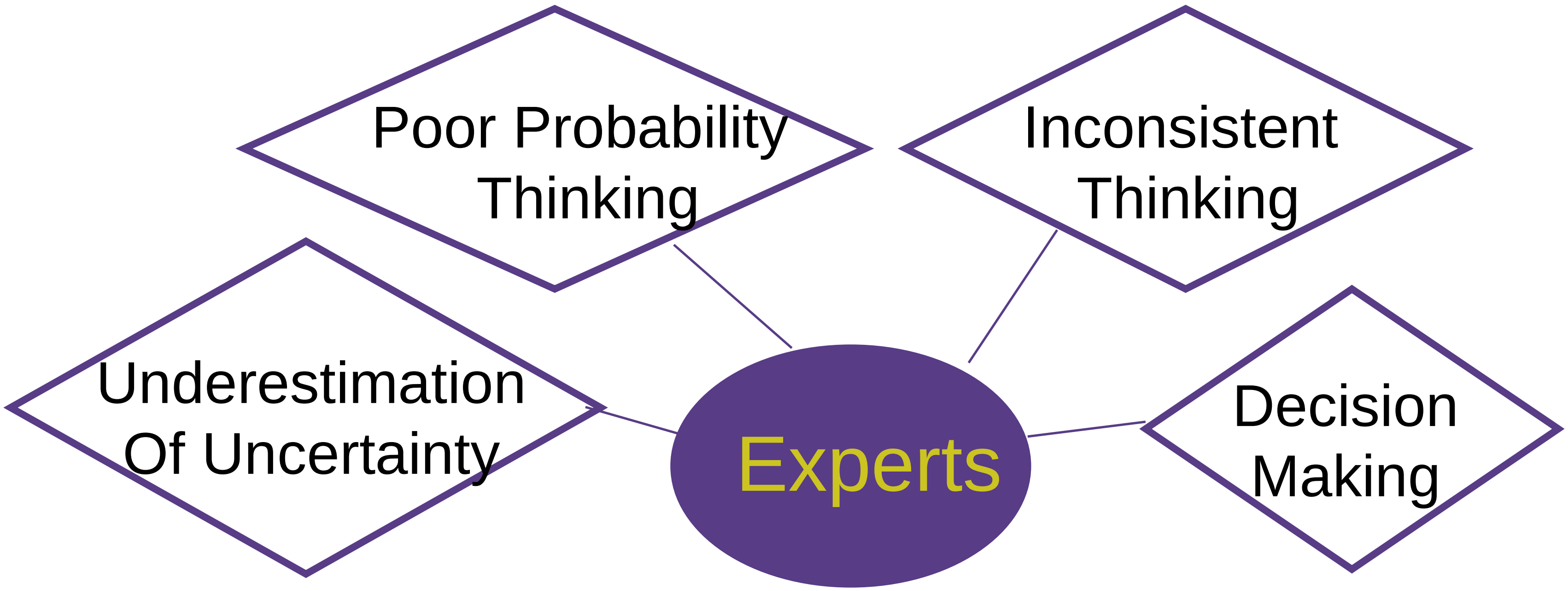
- **Underestimation of uncertainty (false precision)**
- Availability (accounting for rare events)
- Anchoring (cannot move from preconceptions)
- Inconsistency (forgetting what preceded)

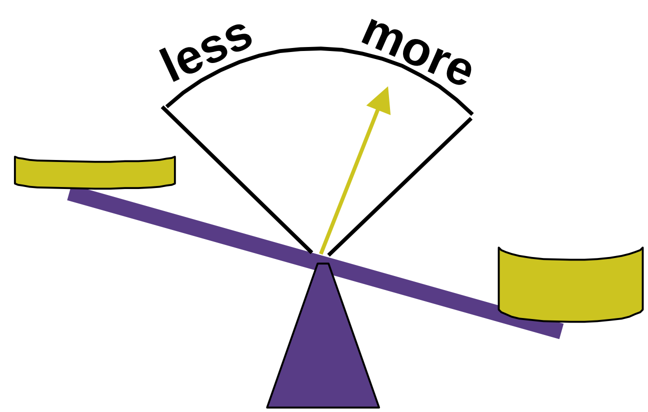


Motivational biases:

- Group think (follow the leader)
- Impression Management (politically correct)
- Wishful thinking (wanting makes it a reality)
- Misrepresentation (bad translation)

Role of Expert Knowledge in Uncertainty Quantification — Contributions to Uncertainty





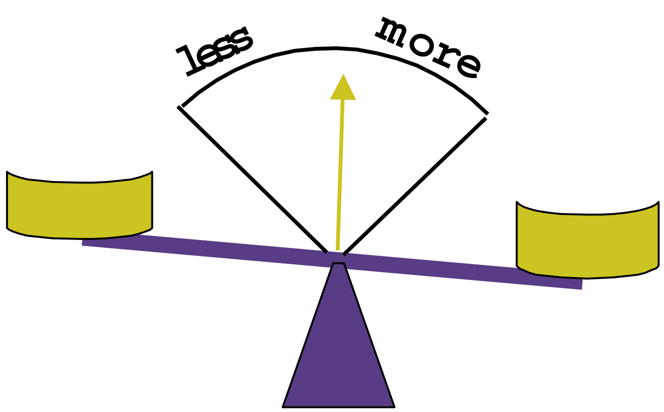
What Tools / Technologies Are Available To Counter These Contributions?

I. Formal, structured elicitation of expertise and expert judgment

- Draws from cognitive psychology, decision analysis, statistics, sociology, cultural anthropology, and knowledge acquisition.
- Counters common biases arising from human cognition and behavior.
- Adds rigor, defensibility, and increased ability to update the judgments.

I. Formal, Structured Elicitation of Expertise and Expert Judgment

- Minimizes biases
- Provides documentation
- Utilizes the way people think, work, and problem solve
- Provides what is necessary for uncertainty quantification:
  - -Sources,
  - -Quantification,
  - -Estimates and Updates,
  - -Methods of propagation



II. Mathematics (Theories) Handling Ignorance, Ambiguity, Vagueness and the Way People Think

- Probability Theory (different interpretations within e.g., Frequentist, Subjective/Bayesian)
- Possibility Theory (crisp or fuzzy set)
- Fuzzy Sets
- Dempster-Schafer (Evidence)Theory
- Choquet Capacities
- Upper and Lower Probabilities
- Convex Sets
- Interval Analysis Theories
- Information Gap Decision Theory (non measure based)

Mathematical Theories — Frameworks for Expert Thinking

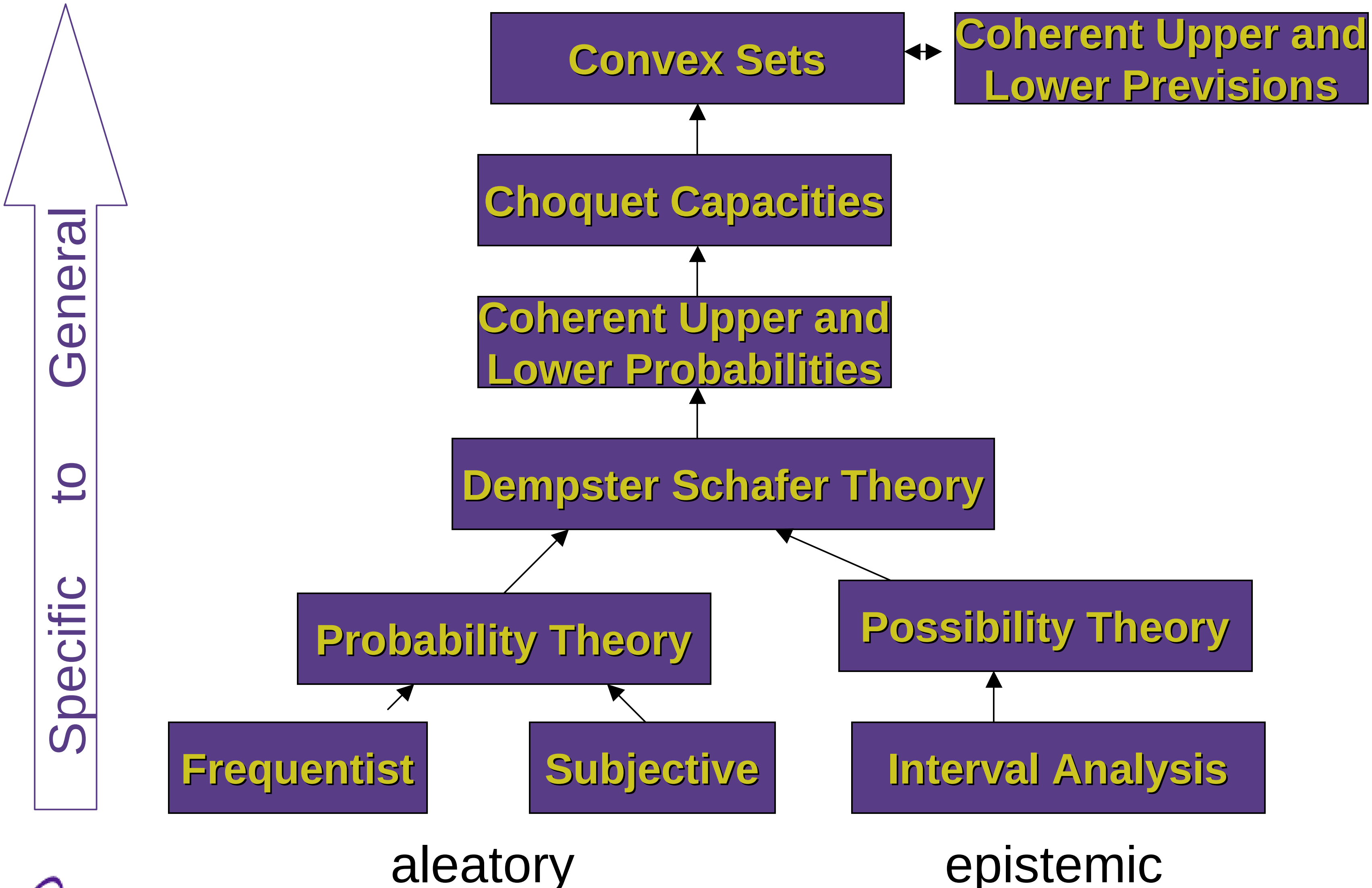
Characteristics

- Set based (crisp or fuzzy)
- Axiomatic
- Calculus (rules for implementing axioms)
- Consistent / coherence
- Computationally practical (??)
- Measure based (not all!)

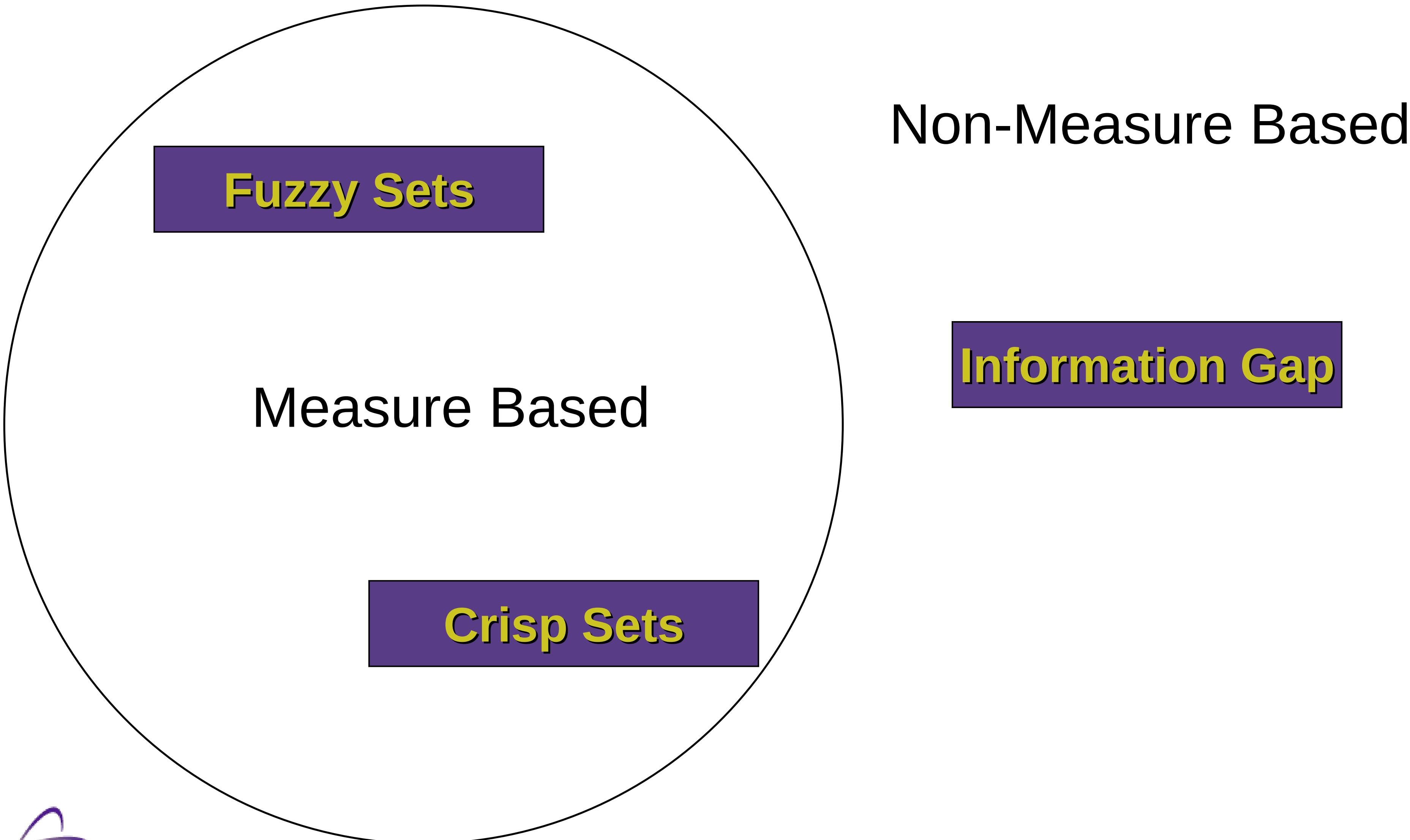
Goal: Provide Metrics for Uncertainty

For combining uncertainties there needs to be a bridge between the various theories.

Hierarchy of Theories for Crisp Sets



Set Based Theories for Uncertainty



Some Measure Theory Approaches

Probability Theory

Based on single measure function (additivity, monotonic)

$$\Pr: 2^X \to [0,1]$$

$$\Pr(\varnothing)=0$$

$$\Pr(X)=1$$

$$\Pr\left(\bigcup_i A_i\right)=\sum_i \Pr(A_i)-\sum_{j<k}\Pr\left(A_j \cap A_k\right)+\cdots+(-1)^{n+1}\Pr\left(\bigcap_i A_i\right)$$

$$\Pr\left(\bigcap_i A_i\right)=\sum_i \Pr(A_i)-\sum_{j<k}\Pr\left(A_j \cup A_k\right)+\cdots+(-1)^{n+1}\Pr\left(\bigcup_i A_i\right)$$

Dempster-Schafer Theory

Based on two measure functions — belief and plausibility (monotonic & nonadditivity)

$$\mathrm{Bel}: 2^X \to [0,1] \qquad \mathrm{Pl}: 2^X \to [0,1]$$

$$\mathrm{Bel}(\varnothing)=0 \qquad \mathrm{Pl}(\varnothing)=0$$

$$\mathrm{Bel}(X)=1 \qquad \mathrm{Pl}(X)=1$$

$$\mathrm{Bel}\left(\bigcup_i A_i\right)\geq\sum_i \mathrm{Bel}(A_i)-\sum_{j<k}\mathrm{Bel}\left(A_j \cap A_k\right)+\cdots+(-1)^{n+1}\mathrm{Bel}\left(\bigcap_i A_i\right)$$

$$\mathrm{Pl}\left(\bigcap_i A_i\right)\leq\sum_i \mathrm{Pl}(A_i)-\sum_{j<k}\mathrm{Pl}\left(A_j \cup A_k\right)+\cdots+(-1)^{n+1}\mathrm{Pl}\left(\bigcup_i A_i\right)$$

Possibility Theory

Based on two measure functions — possibility & necessity (monotonic & nonadditivity)

$$\mathrm{Pos}: 2^X \to [0,1] \qquad \mathrm{Nec}: 2^X \to [0,1]$$

$$\mathrm{Pos}(\varnothing)=0 \qquad \mathrm{Nec}(\varnothing)=0$$

$$\mathrm{Pos}(X)=1 \qquad \mathrm{Nec}(X)=1$$

$$\mathrm{Pos}\left(\bigcup_i A_i\right)=\sup_i \mathrm{Pos}(A_i)$$

$$\mathrm{Nec}\left(\bigcap_i A_i\right)=\inf_i \mathrm{Nec}(A_i)$$

Potential Uncertainty Metrics

- Hartley measure for nonspecificity

$$H(A) = \log_2 |A| , \quad |A| \text{ is cardinality of } A$$

- Generalized Hartley measure for nonspecificity in DST

$$N(m) = \sum_{A \in 2^X} m(A) \log_2 |A|, \quad m: 2^X \to [0,1] \quad m(\varnothing) = 0, \sum_{A \in 2^X} m(A) = 1$$

- U-uncertainty measure for nonspecificity in possibility theory

$$U(r) = \sum_{i=2}^{n} (r_i - r_{i+1}) \log_2 i, \quad r(x) = \mathrm{Pos}(\{x\}) \quad r_i \geq r_{i+1} \forall i$$

- Shannon entropy for total uncertainty in probability theory

$$S(p) = -\sum_{x \in X} p(x) \log_2 p(x)$$

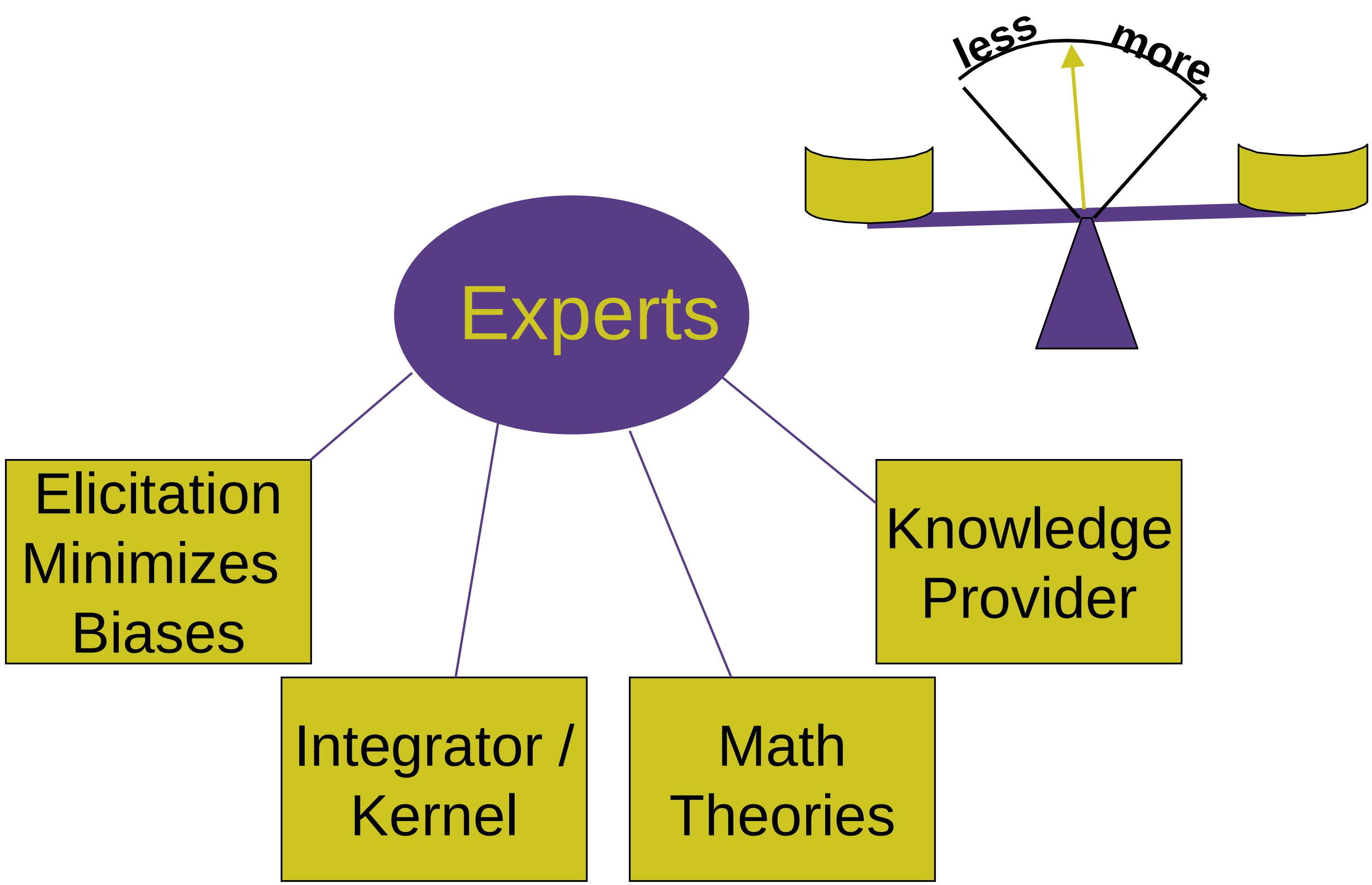
- Generalized Shannon entropy for total uncertainty in DST

$$AU(\mathrm{Bel}) = \max_{p_x} \left(-\sum_{x \in X} p_x \log_2 p_x \right), \quad Bel(A) \leq \sum_{x \in A} p_x \quad \forall A \in 2^X$$

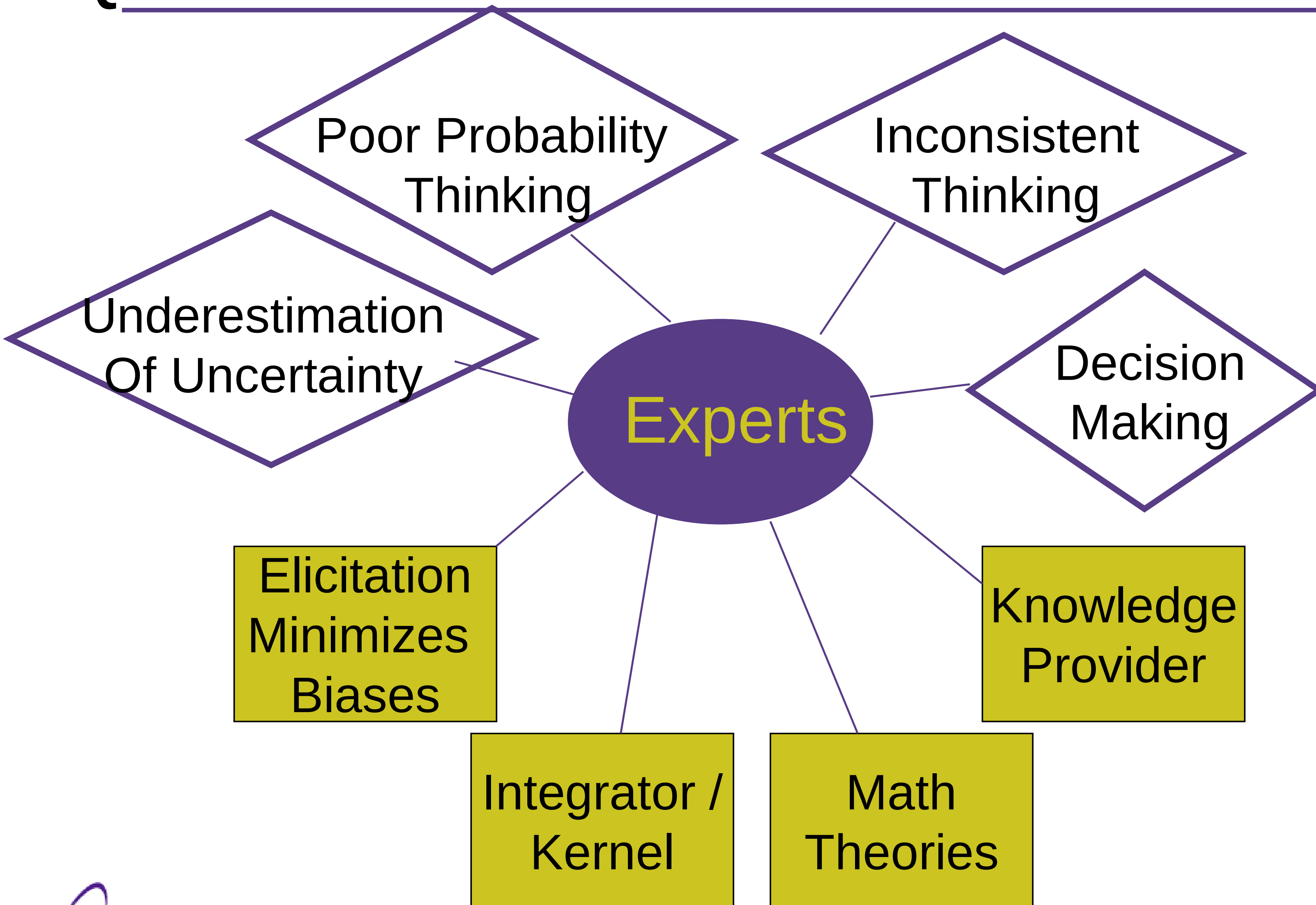
- Hamming distance for fuzzy sets

$$f(A) = \sum_{x \in X} \left[1 - |2A(x) - 1| \right], \quad A(x) \text{ is membership function}$$

Role of Expert Knowledge in Uncertainty Quantification — Gains Understanding



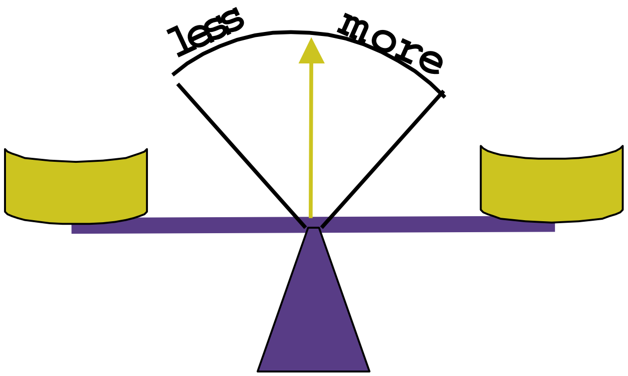
Role of Expert Knowledge in Uncertainty Quantification — Contributions & Understanding



Role of Expert Knowledge in Uncertainty Quantification

Are We Adding More Uncertainty or More Understanding?

A question of balance.



With proper elicitation methods and alternatives probability theory for uncertainties, experts can provide the information,estimation, and integration necessary for understanding uncertainty.