Reducing Error in Estimating Production Costs of Multiple Unit Procurements

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Presentation Topics

- Learning Curve models.
- Developing Learning Curve parameters.
- Shifting Reference Point from T1.
- How to select appropriate Reference Point.
- Demonstration of improvement achieved.

Standard Practice for Estimating Costs of Multiple Unit Procurements

- Apply a cost improvement or "Learning Curve" (LC) rate to account for improvements in:
  - Management
  - Engineering processes
  - Production efficiency
- Experience has shown that unit costs decrease (although at a declining rate) during the production process - regardless of how long the production runs.
- There are two predominate schools of thought on how to apply LCs to estimate production cost.
  - Cumulative Average Unit Cost Theory
  - Unit Cost Theory

Cumulative Average Unit Cost Theory

- Cumulative Average Unit Cost (CAUC) Theory posits that CAUC of successive production units decrease at a constant rate each time the production quantity is doubled.
- That constant rate is referred to as the "CAUC Learning Curve Slope" and is often expressed as a percent (e.g., 90%).
- Standard form of the CAUC Theory equation is

  $Y = ax^b$, where:

  Y is Cumulative Average Unit Cost of x units.

  a is theoretical first unit cost (T1 or CAUC1).

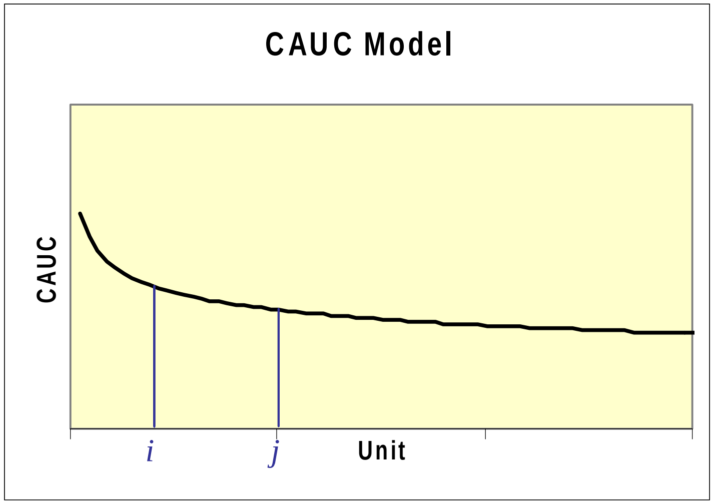
  b is learning curve exponent, $b = \frac{\ln(slope)}{\ln(2)}$

  x is production quantity.

Total Production Costs using CAUC

- Total Production Cost (TPC) for x units

$$TPC = ax^b x = ax^{b+1}$$



- Lot Total Cost (LTC)

$$LTC = a(j^{b+1} - i^{b+1})$$

where j is last unit of lot in question and i is last unit of prior lot.

- Lot Average Cost (LAC) can be determined by dividing LTC by Lot Quantity (q).

$$LAC = \frac{LTC}{q}$$

Unit Cost Theory

- Unit Cost Theory posits that unit costs of production units decrease at a constant rate each time production quantity is doubled.
- That constant rate is referred to as the "Unit Learning Curve Slope" and is often expressed as a percent.
- The standard form of the Unit Cost Theory equation is very similar to the CAUC model.

  $Y = ax^b$, where:

  Y is Unit Cost of the x^{th} unit.

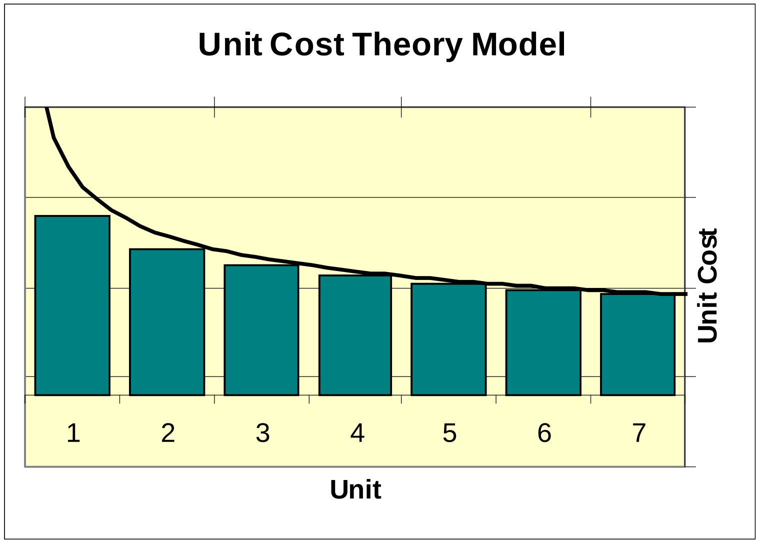
  a is theoretical first unit cost.

  b is learning curve exponent, $b = \frac{\ln(slope)}{\ln(2)}$

  x is number of units produced.

Total Production Costs using Unit Theory

- Unit Theory is discrete so Lot Total Cost may be determined by summing unit costs for each unit



$$LTC = \sum_{x=i}^{x=j} ax^b$$

- It is often convenient to estimate lot costs using a continuous approximation of the discrete distribution.

$$LTC = \int_{i-.5}^{j+.5} ax^b dx = \frac{a\left((j+.5)^{b+1} - (i-.5)^{b+1}\right)}{b+1}$$

Where i is first unit and j is last unit of the lot

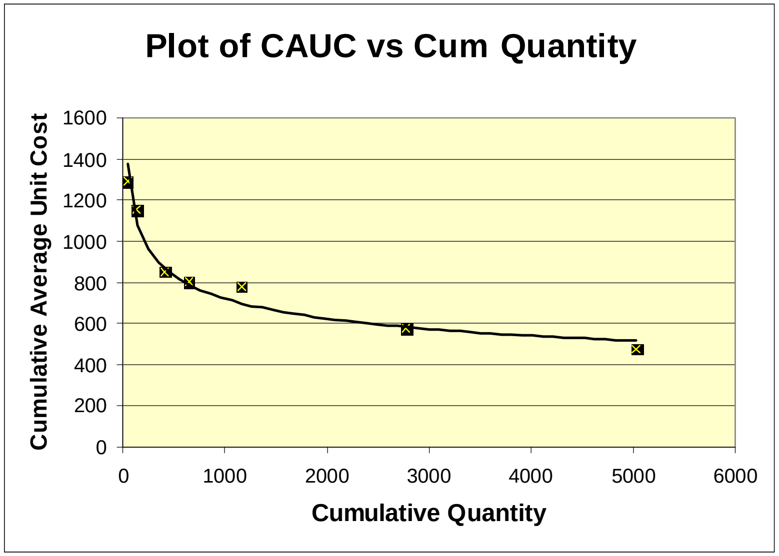
Developing a CAUC Model from Actual Data

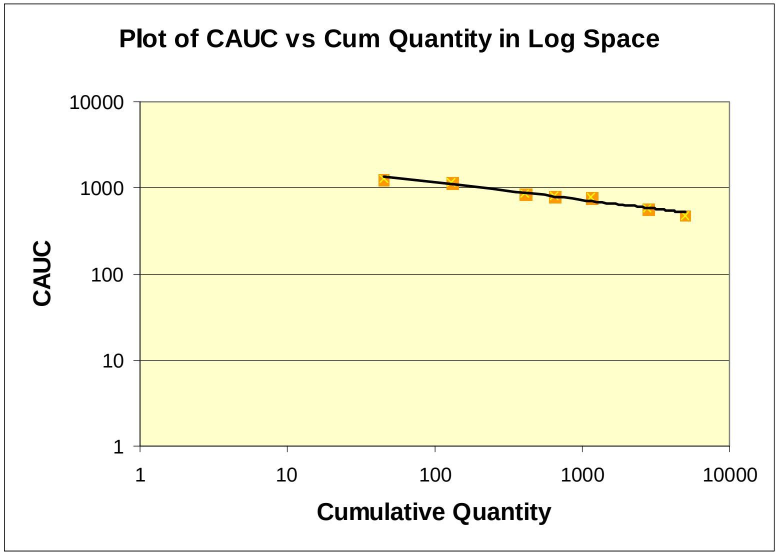
- T1s and Learning Curve slopes can be derived from historical production costs. Here is a sample data set. (“Cum” = Cumulative)

Lot	BQ	Cost	Cum Q	Cum C	Cum AUC
1	45	58000	45	58000	1289
2	75	89000	120	147000	1225
3	276	195000	396	342000	864
4	230	175000	626	517000	826
5	509	375000	1135	892000	786
6	1618	695000	2753	1587000	576
7	2253	805000	5006	2392000	478

CAUC is the dependent variable.

Cum Quantity is the independent variable.





Curve Fitting Model

Iteratively Re-weighted Least Squares (IRLS) Analysis:

- Minimizes the sum of squared percentage error:

$$\sum \left(\frac{actual_i - pred_{ij}}{pred_{i(j-1)}} \right)^2$$

- May be performed using Excel Solver.

Our first iteration uses values for a and b derived from a log/log regression model to determine $pred_{i(j-1)}$ and then finds values of a and b that minimize the squared percent error function. We then iterate this process, each time retaining our previous predictions in the denominator, until differences between our new predictions and previous predictions approach 0.

IRLS has several desirable properties vis-à-vis log/log regression:
- The minimization function is in unit space (vice log space).
- Weights each data point equally.
- Percent bias approaches 0.

The “Covariance Matrix”

- The covariance matrix can be developed as shown

$$Var(\beta^*) = \lambda \left| \left(\sum_{i=1}^{n} \frac{\left[J_i^T J_i \right]}{f(x_i, \beta^*)^2} \right)^{-1} \right|$$

$$where\ J_{n\times m} \equiv \left| \partial f(x_i, \beta^*) / \partial \beta_j \right|_{n\times m}$$

$$and\ \lambda = \frac{1}{n-m} \sum_{i=1}^{n} \left(\frac{yi - f(x_i, \beta^*)}{f(x_i, \beta^*)} \right)^2$$

- Adaptation of Dr. Matthew Goldberg’s presentation at DODCAS 1999.

The covariance matrix enables us to develop variability parameters for the IRLS coefficients.

Model Results Using IRLS

	Estimate	SE
T1	3165.5	340.3
b	-0.213	0.0164
Slope	86.3%	

11%

Using our sample data from Chart 8.

One Standard Deviation Interval		
	Low	High
T1	2825	3506
Slope	85.3%	87.3%

Nice tight interval on Learning Curve Slope, but T1 value has wider variability.

% SE	7.95%
% Bias	-0.005%

Cost Formula: $CAUC_n = 3165.5 * n^{(-0.213)} * error$

Variability parameters tell us how well this equation predicts CAUC for historical system, *but additional sources of variability are introduced when predicting cost of a new system.*

Estimating Costs of a New System

- Learning Curve Slope (b). Typical methods for estimating Learning Curve Slope include:
  - Analogy to another program
  - Average of several similar programs
  - Analyst Assumptions or “Expert Judgement”
- First Unit Cost (a). Typical methods for estimating First Unit Cost include:
  - Cost Estimating Relationship derived from historical data on earlier programs..
  - Analogy to another program.
  - Derived from prior data from same program.
- Annual Production Quantities (x). Usually determined by mission requirements and availability of procurement funding.

- **So, how good are these methods? Let’s look first at some Learning Curves derived from historical data.**

Learning Curve Slopes for Missile Programs

- Study of missile programs shows that CAUC Learning Curve Slopes developed from historical data range widely.
  - Slopes derived for 13 historical programs range from 95.8% to 75.5%, with median slope of 84.0%
  - Ranges only slightly narrowed when stratified by contractor, developing service, missile type, or first year of manufacturing.
  - When stratified using multiple categories, sample sizes are too small for analysis.
- Therefore cost analysts tend to look for closest analogy using multiple criteria - but we don’t know how close the analogy fits the new program.
- Let’s look at how much a cost estimate can be affected by choice of learning curve.

Impact of Learning Curve Choice

- Let's assume we know that first unit cost of a new missile is exactly \$1.0M, and the production requirement is 5000 missiles.
- Let's accept the median historical LC slope (84%) for our pre-production cost estimate, so that $b = \ln(.84)/\ln(2) = -.252$
- Total production estimate for 5000 missiles is then

  $$Prod\ Cost = 5000\,CAUC_{5000} = (1.0)(5000)^{1-.252} = \$586.9M$$

- But if, when production begins, the contractor is only able to manage a 90% learning curve slope, and $b = \ln(.90)/\ln(2) = -.152$

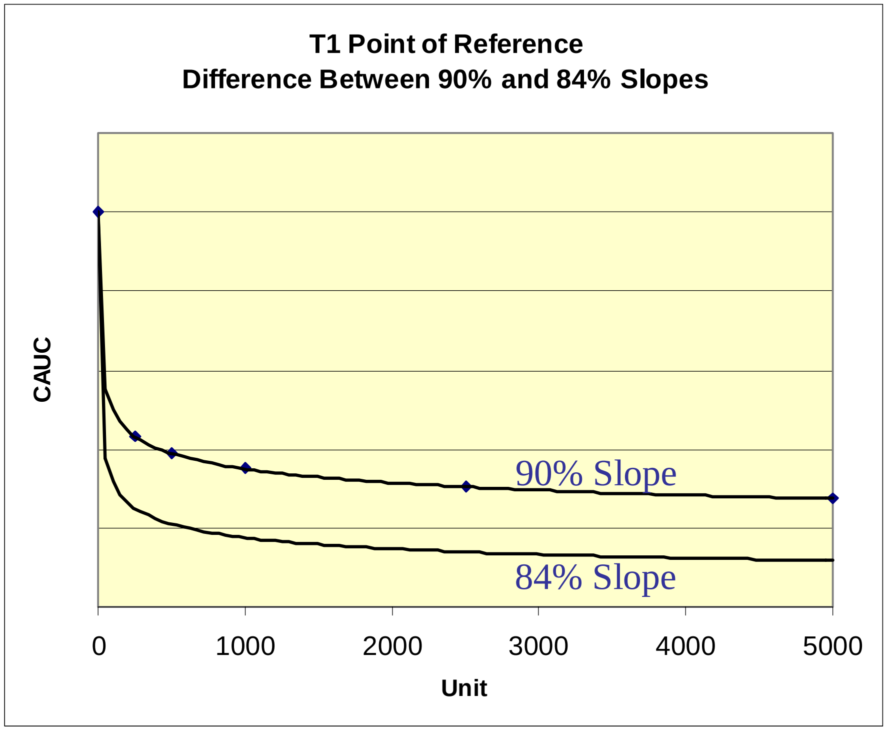
  the actual production cost is now

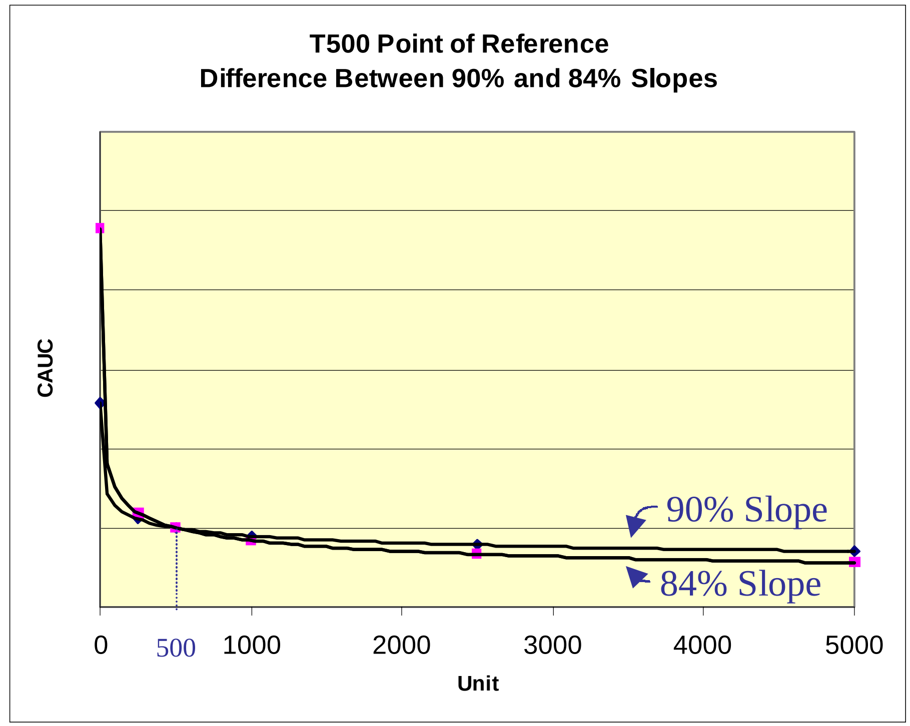
  $$Prod\ Cost = 5000\,CAUC_{5000} = (1.0)(5000)^{1-.152} = \$1370.0M$$

- **So we underestimated total production cost by <u>\$783M</u>!**
- This error is too large, <u>even for DoD cost estimates</u>.
- ***How can we improve the estimate?***

Mitigating Effects of Learning Curve Choice

Estimating production costs based on T1 magnifies any error we make in selecting an appropriate learning curve. The charts below illustrate how much better the estimate can be if we move the reference point away from T1.

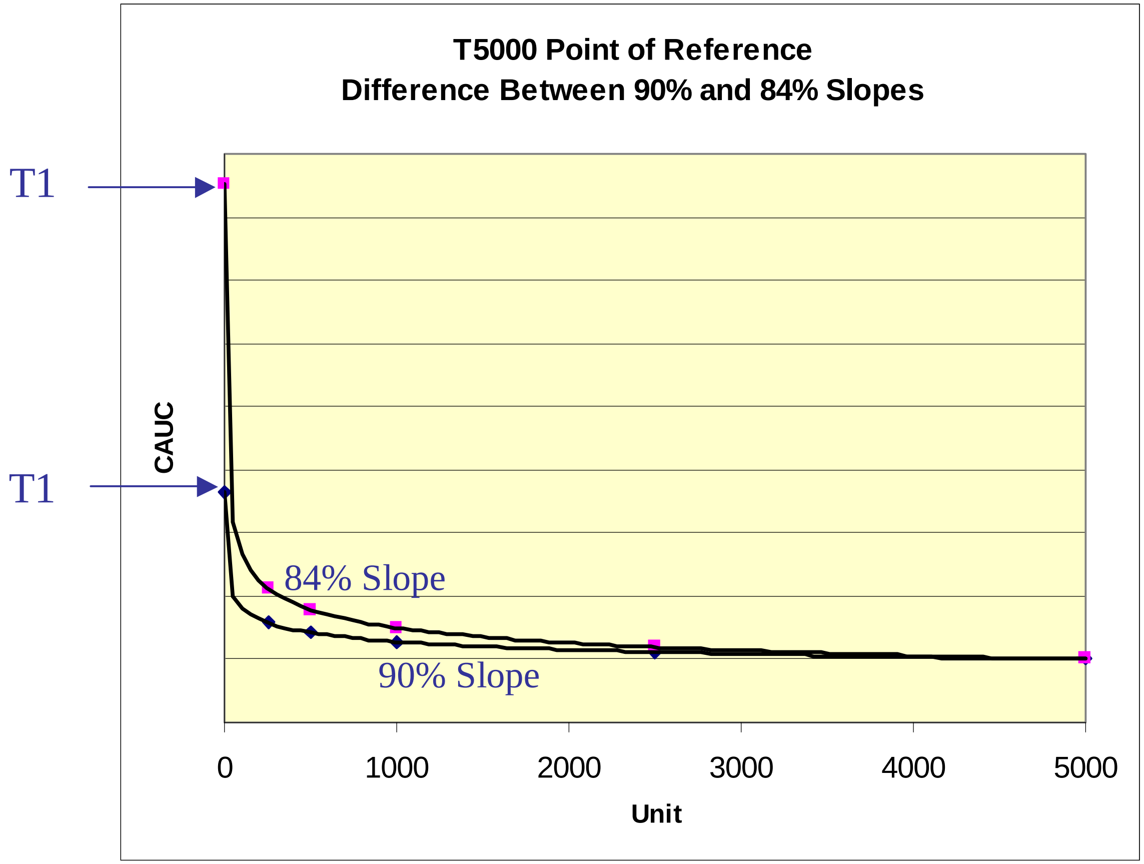




As we move our reference point to the right towards 5000 units, the cost estimate is impacted less and less by a wrong choice in learning curves.

What if the Reference Point is the Delivery Quantity?

- The total production cost estimate will not be impacted by choice of learning curve.
- But we risk large errors in the cost estimate of early production lots, and this will cause budgeting problems.



Proposed Model Standard Form

$$CAUC(N) = T_m (N / m)^b$$

- N is the number of production units.
- m is the Cost Reference Point.
- T_m is the CAUC of m units.

- For fitting a learning curve model:
  - Dependent variable is CAUC(N)
  - Independent variable is (N/m)
  - Analysis of data produces estimates for T_m and b.

Selecting a Good CAUC Reference Point (CRP)

- Desirable Characteristics:
  - Mitigates effect of choosing wrong learning curve for both
    - Production Total Cost
    - Annual Production Costs
      - Somewhere between T1 and Total Delivery Quantity
  - Robust for use with multiple programs - doesn't exceed total production requirement of most programs.
  - ***Least possible error in estimating the CAUC at the Reference Point.***

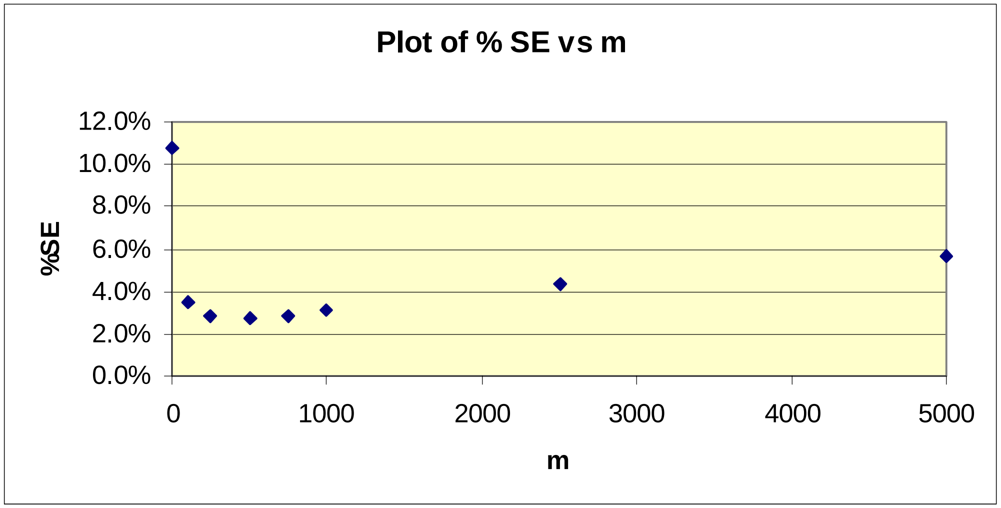
Least Possible Estimating Error is Important

- Cost Estimating Relationships (CERs) are influenced by Tm estimating error.
  - CERs use physical properties or characteristics of systems to predict cost.
  - They start with development of Tm estimates for several similar systems from historical data using learning curve models (usually at T1).
  - They use these Tm estimates as dependent variables in regression models - usually assuming that the Tm values are known with certainty.
  - Error in estimating Tm for the historical systems degrades the accuracy of the CER.
- If we use an analogy, the Tm is derived from a learning curve model for the analgous system.
- ***So, if we minimize the error in developing the Tm in our learning curve models, we improve the accuracy of our cost estimating reference point.***

Minimizing Error in Estimating Tm

- Selecting an appropriate value for “m” can be done by examining % SE of Tm estimates at various values of “m”.
- The table and chart below (based on the data in chart 8) show that % SE is minimized near T500, and every value has lower % SE than T1.
- In support of BMDO, we use T250 for missile programs based on relatively small %SE and the anticipated procurement quantities of BMD missile systems.

m	Tm	SE(Tm)	%SE(Tm)
1	3165.5	340.3	10.8%
100	1189.0	42.0	3.5%
250	978.3	28.2	2.9%
500	844.2	23.1	2.7%
750	774.4	22.3	2.9%
1000	728.4	22.6	3.1%
2500	599.3	26.3	4.4%
5000	517.1	29.4	5.7%



Variability in Production Estimates Using T1

- Let’s assume missile learning curves are triangularly distributed between 95.8% and 75.5%, with a most likely value of 84%.
- Let’s assume our T1 is lognormally distributed with a Point Estimate of 1 and a 30% SE.
- Now let’s randomly select a learning curve and T1 from their respective distributions 5000 times in a simulation to model the distribution of production cost outcomes.

Here are the simulation results :

Statistic	Value
Trials	5,000
Mean	815.00
Median	652.44
STD	577.47
Skewness	1.79
Kurtosis	7.33

Percentile	$M
10%	**276.04**
30%	459.26
50%	652.44
70%	928.32
90%	**1,585.77**

The 10-90% Range is $1,309M.

- **Wouldn’t you like to have an estimating methodology that narrows the range of probable outcomes better than this?**

Variability in Production Estimates Using T250

- Let's continue to assume missile learning curves are triangularly distributed between 95.8% and 75.5%, with a most likely value of 84%.
- Let's assume our T250 is lognormally distributed with a Point Estimate of .25 and a 25% SE (Assumes that better knowledge of dependent variable gives us a 5% reduction in %SE in the CERs as shown on chart 19).
- Now let's simulate the production run 5000 times.

Here are the simulation results :

Statistic	Value
Trials	5,000
Mean	535.11
Median	506.59
STD	176.49
Skewness	0.92
Kurtosis	4.14

Percentile	$M
10%	**336.69**
30%	425.01
50%	506.59
70%	602.59
90%	**773.36**

The 10-90% Range is $337M, down almost $1,000M from a T1 Cost Estimate.

- **The T250 Reference Point substantially reduces the risk associated with our cost estimates.**

Summary

- *T1 is a poor reference point from which to start an estimate.*
  - *It magnifies the impact of errors in selecting a learning curve slope.*
  - *The SE in estimating a T1 is much larger than the SE for estimating Tm, where 1 < m < very large number.*
- *Using Tm as a reference point improves the accuracy the estimate.*
  - *Error in selecting a learning curve is mitigated.*
  - *Estimates of Tm are more precise and less influenced by the value of the learning curve slope.*
  - *Provide a better basis for CER development.*

Don't Use T1s in Cost Estimates