Presentation on How Bayesian Reliability Analysis was Developed and Implemented for Production Decisions

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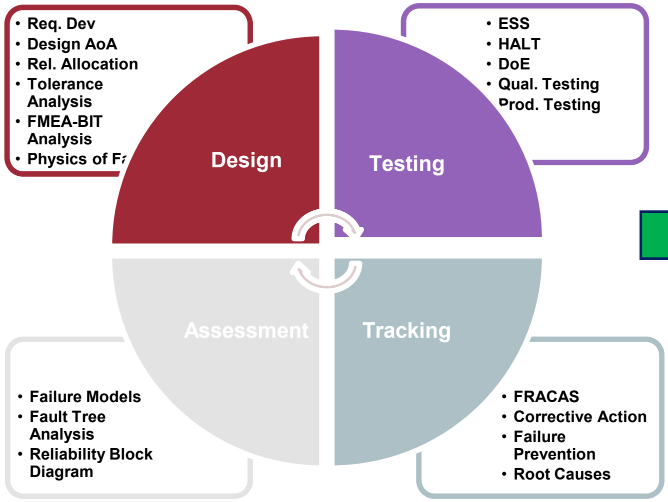
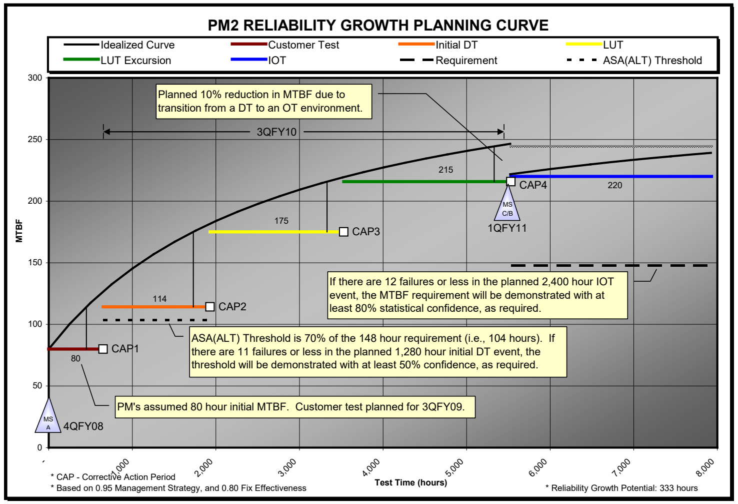
Key Program Evolution

- **Clearly Identifying and Characterizing the Transition from Design for Reliability (DFR) to System Level Reliability Growth Testing (RGT) is a Critical Point in the Reliability Growth Program (RGP) Towards a Mature System**





Diagrams are Referenced from AMSAA Briefings

DFR **RGP**

Evolution from Design Stage to a Mature System

Key Program Milestone - Production Decision

- **Must Establish That the Program is Reaching Expected Maturity Levels of Functional Reliability (FR) Using DFR and RGT as a Part of the RGP**
- **Reliability Growth Planning Models (Discrete)**
  - **Focus on Relationship of Initial to Mature Reliability**
  - **At the Production Decision Current FR Should be 25 – 50% of the Range Between Initial and Mature FR**
  - **Accurate Evaluation of Current Reliability is Essential**
  - **To Ensure Sufficient Tests to Mature in Early Production**
  - **However, There is Very Little System Level Testing Available**
  - **Need to Supplement with Other Test Data and Analysis**
- **Lends Itself to a Bayesian Approach**

The significant question early in a program at the production decision point - is the system on track to meet its reliability requirements?

Why a Bayesian Approach?

- **Do not want to underestimate nor over estimate reliability early in evaluation & test cycle.**
  - **Protected against "perfect" assessment early on.**
  - **Protected against low estimates if there are early failures.**
- **Want to make use of previous data on similar systems and expert opinion as to component, subsystem and system reliabilities.**
- **Want reliability to converge to that given by actual test data as more and more data becomes available.**
- **Want a method that correctly estimates actual "confidence" for the reliability. Classical confidence intervals do not give interval of interest.**

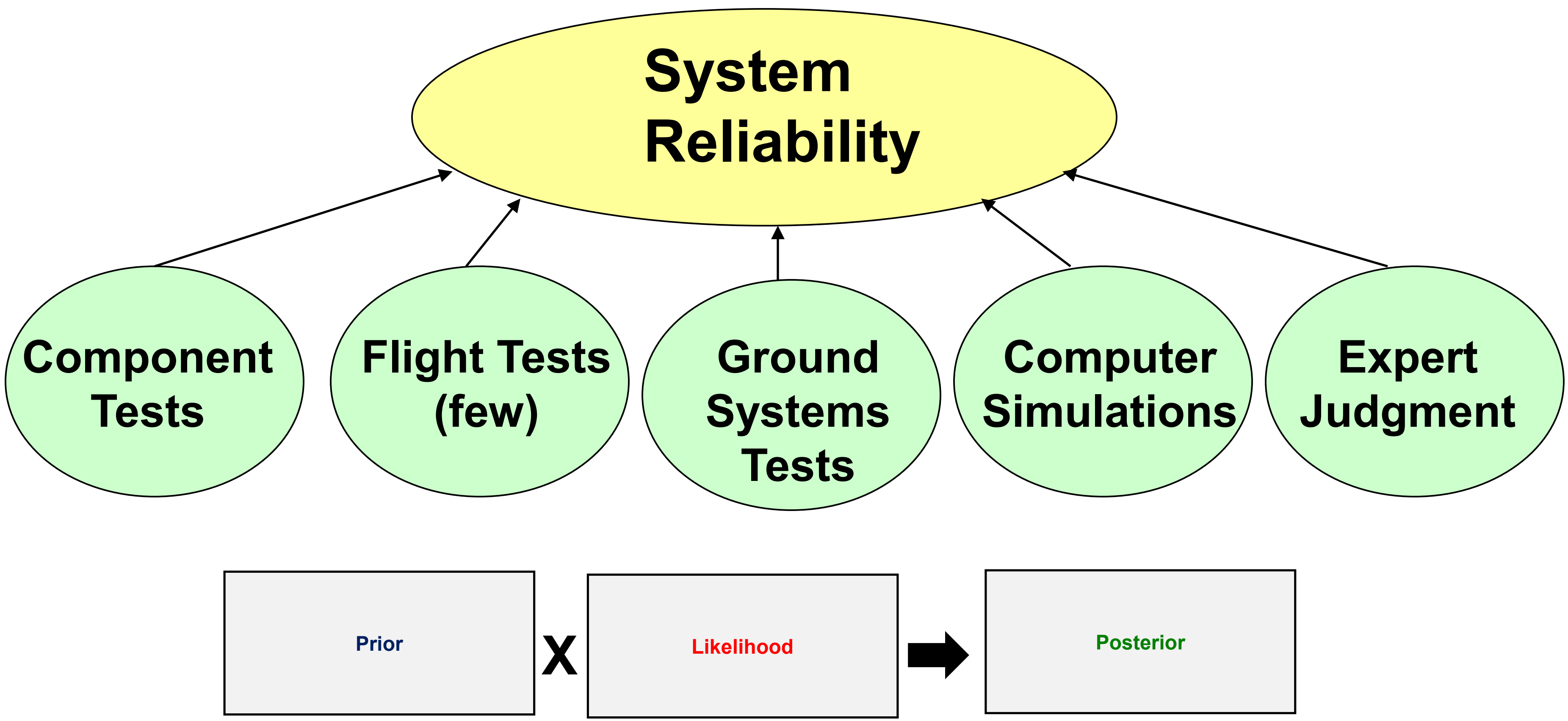
It's not so important that we have an exact point estimate of our reliability, as it is that we have a method of measuring the confidence we have in meeting our decision criteria.

Bayesian made simple

- **We want to <u>combine</u>**
  - **previous (prior) knowledge about our belief on the reliability of the system, $f_{prior}(R)$**
  - **with data from experiments / tests (likelihood) on the subsets of the components of the proposed system, L(data|R)**
- **Produce a (posterior) probability distribution for the present system's reliability, $f_{posterior}(R|data)$,**

General Idea

Investigate methods to credibly combine information from different sources



Model: Priors

- The component priors have been created either from component test data or SME inputs and used to specify the values for nprior[i] and rprior[i] that are used in a beta distribution e.g.

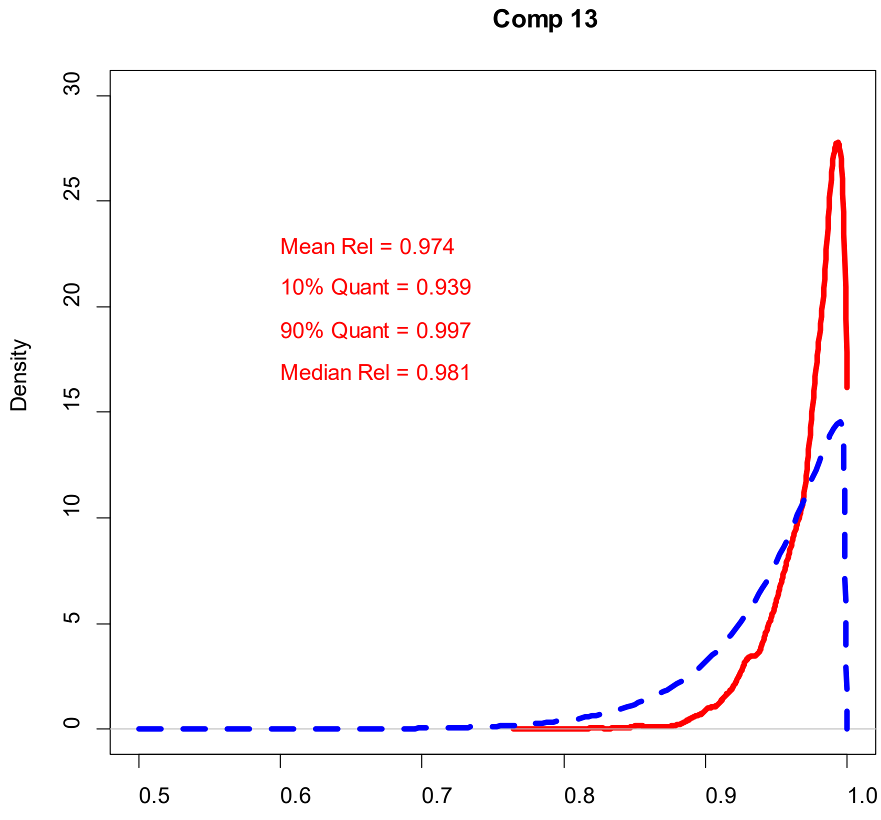
rprior is the mode of the prior distribution and nprior is a weighting (or importance) factor . nprior=0 gives a uniform prior.

$$f_{prior}(R)=Constant^{*}\ R^{(nprior^{*}rprior)}\ {}^{*}\ (1-R)^{(nprior^{*}(1-rprior))}$$

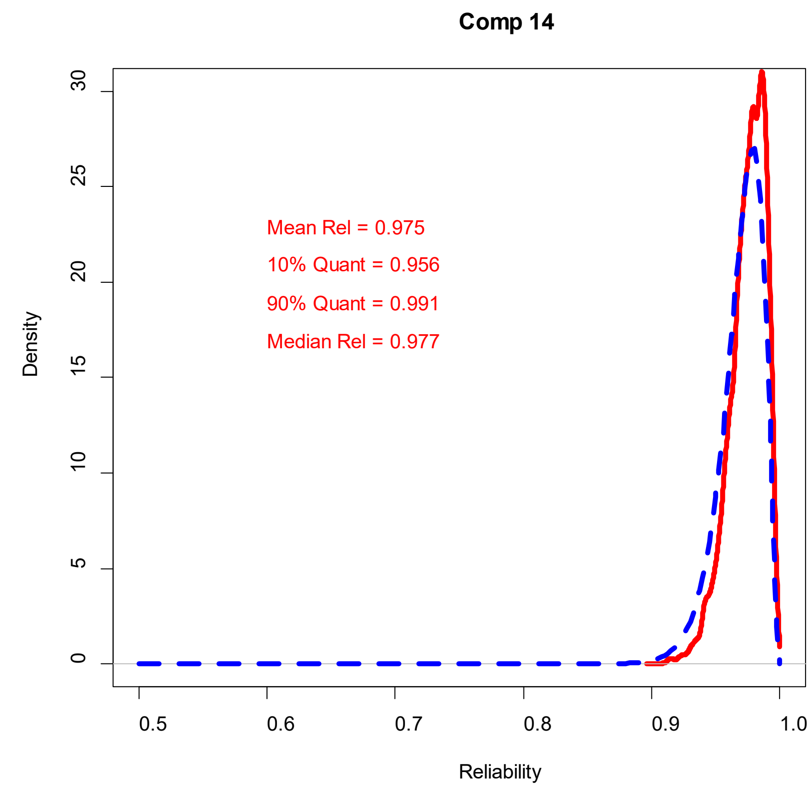
- Beta distributions are versatile and allow for many useful types of prior inputs including 1) pass/fail data from previous tests of components, 2) SME input on mean of component reliability (rprior) and accuracy of estimate (nprior).
- Narrower Peaked - Strong Prior Where Data Supported
- Wider Flat – Weak Prior for SME Inputs

DATA

13 & 14



rprior[13]	19/20
nprior[13]	20



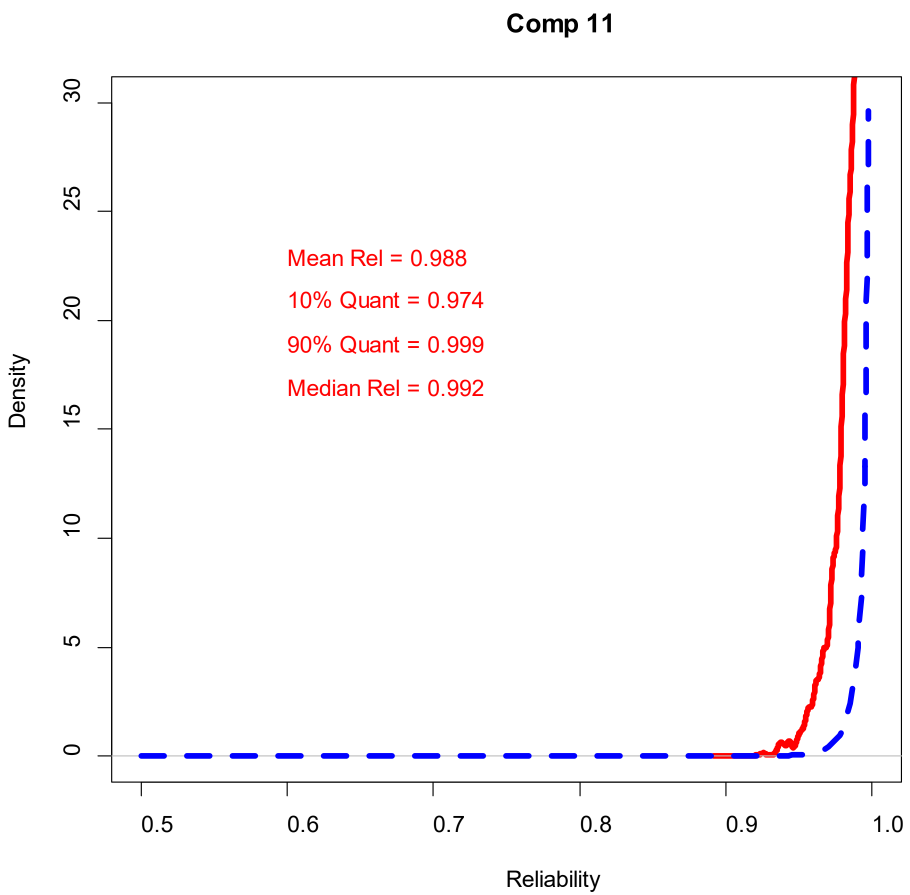
rprior[14]	99/102
nprior[14]	102

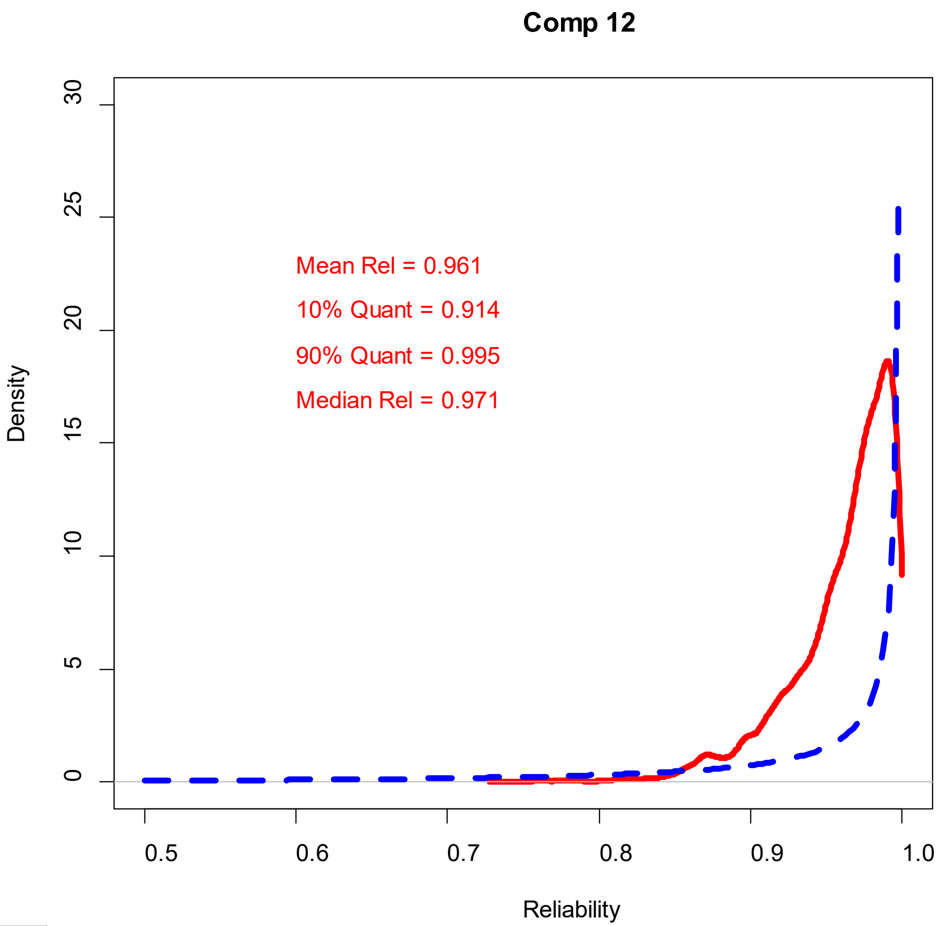
Blue= prior

Red= Posterior

SME

11&12





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rprior[11]	0.99993
nprior[11]	3

rprior[12]	0.99993
nprior[12]	3

Model: Likelihood

- **The likelihood function evaluated using system level test data**
  - **Independence maintained**
- **System involves many discrete functions – binomial distribution selected in the following form**

$$L = \prod_i L(R_i \mid s_i, n_i) \propto \prod_i R_i^{s_i} (1 - R_i)^{n_i - s_i}$$

$$\ln(L) \propto \sum_i \left[s_i \ln(R_i) + (n_i - s_i) \ln(1 - R_i) \right]$$

R = Reliability
n = number of tests
s = number of successes
(n-s) = number of failures

- **The likelihood function for the system during its development phase is a complex product of reliabilities from tests of various subsets of components**
  - **Usually about 20-25 tests that have been conducted at the production decision point.**
  - **No simple formula for the likelihood as each of the tests at the "system level" tested various subsets of components.**

Spreadsheet.dat

Column 1 is P/F for System

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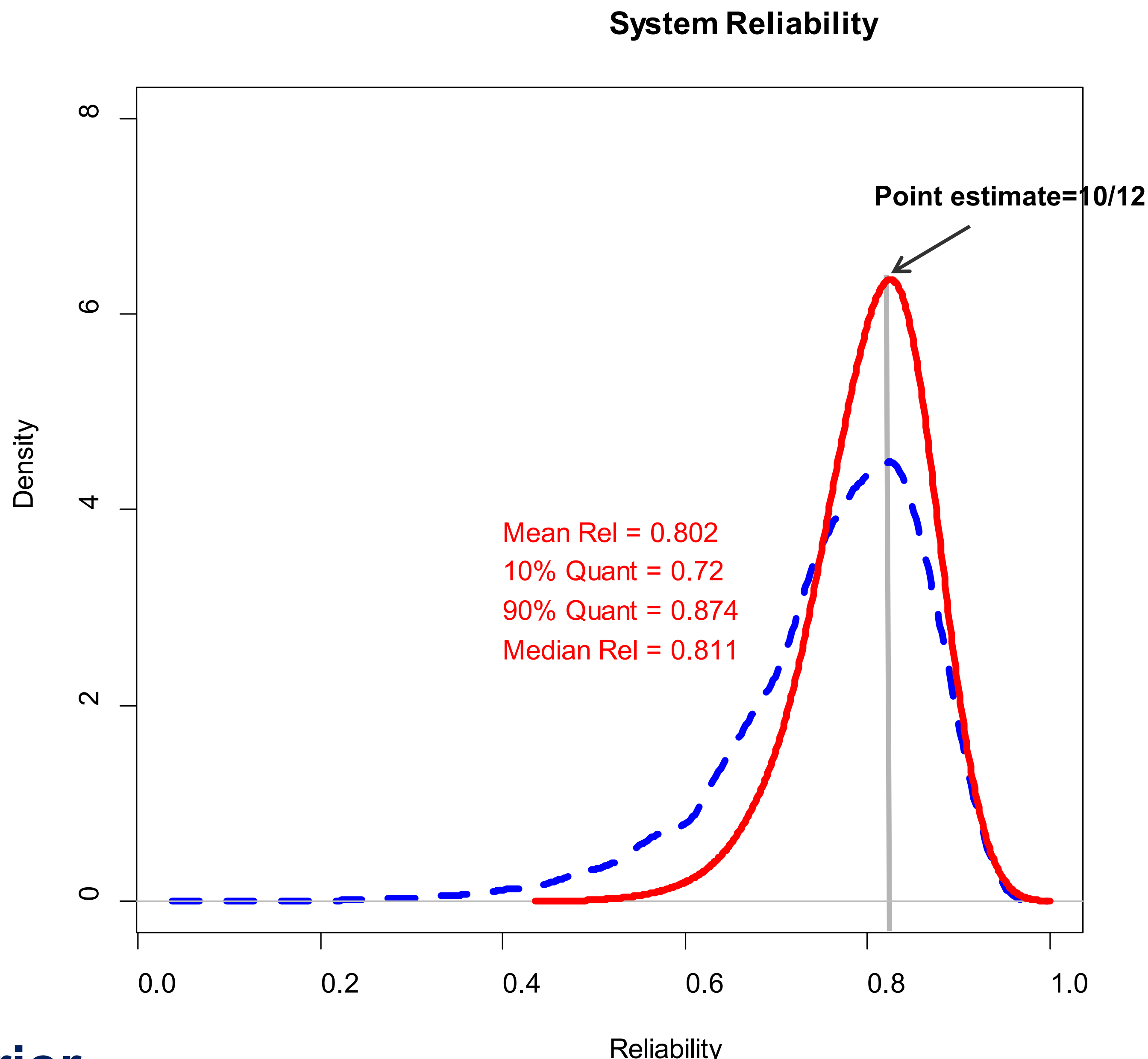
Columns 2 – 25 are P/F or NT(no test not included in test) or NT-F for components that were part of test but were not tested due to component failing upstream.

Model: MCMC Computation

- **Computation was performed using successive substitution MCMC. The updates were all performed using Metropolis**
- **Metropolis - Hastings updates were done with beta proposal distributions centered at the previous value. The acceptance probability was adjusted to reflect the asymmetric proposal density.**
- **The posterior results come from running 100,000 iterations and "burning" the first 10,000 iterations. This appeared sufficient as higher numbers of iterations and various burn values were tried and produced no significant differences in results. (differences in 3rd decimal place)**

Model: Posterior Distribution

- The system reliability has a Mean Reliability = 0.80 and a useful 80% credibility interval [0.72,0.87]
  - Generally consistent with "classical" point estimate – 10/13 = 0.77 but the confidence interval is much smaller than the "classical" [0.56, 0.91]
  - AMSAA Demonstrated Growth Value = 0.83 is consistent with a reasonably conservative part of the credible interval [0.74- 0.78]
- The Bayesian analyses includes the complex nature of the 20-25 "system level " tests most of which do not include all components of the system
- Since there was little useful information available from the time-to-failure tests (i.e. there were no failures over entire time of test), all data was treated as pass/fail in the analyses



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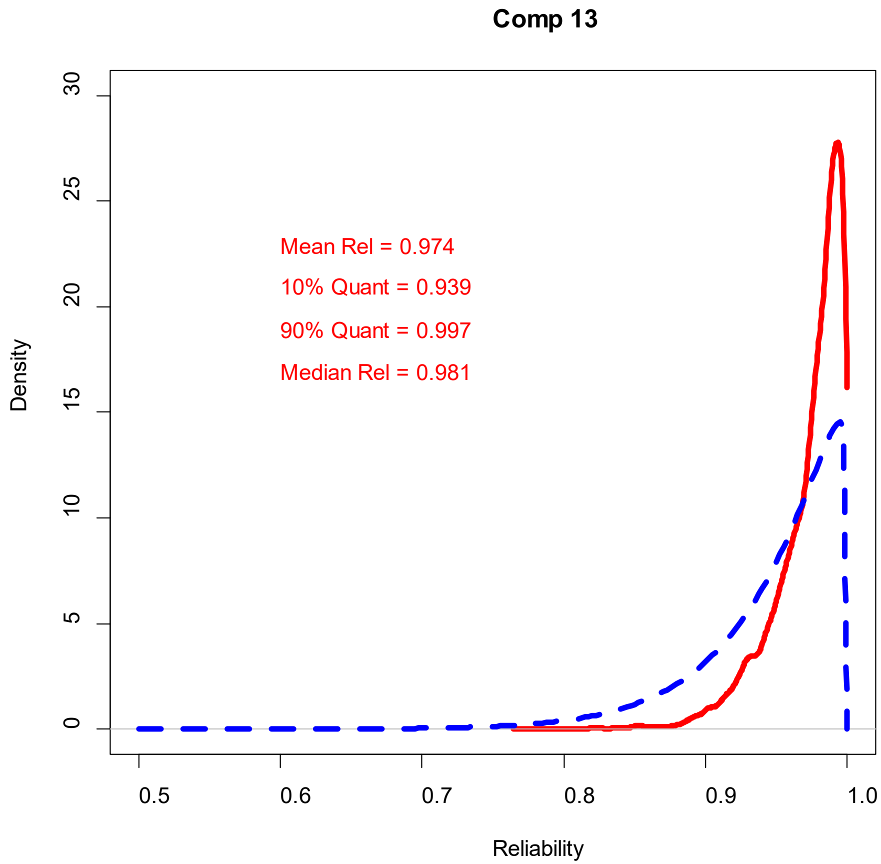
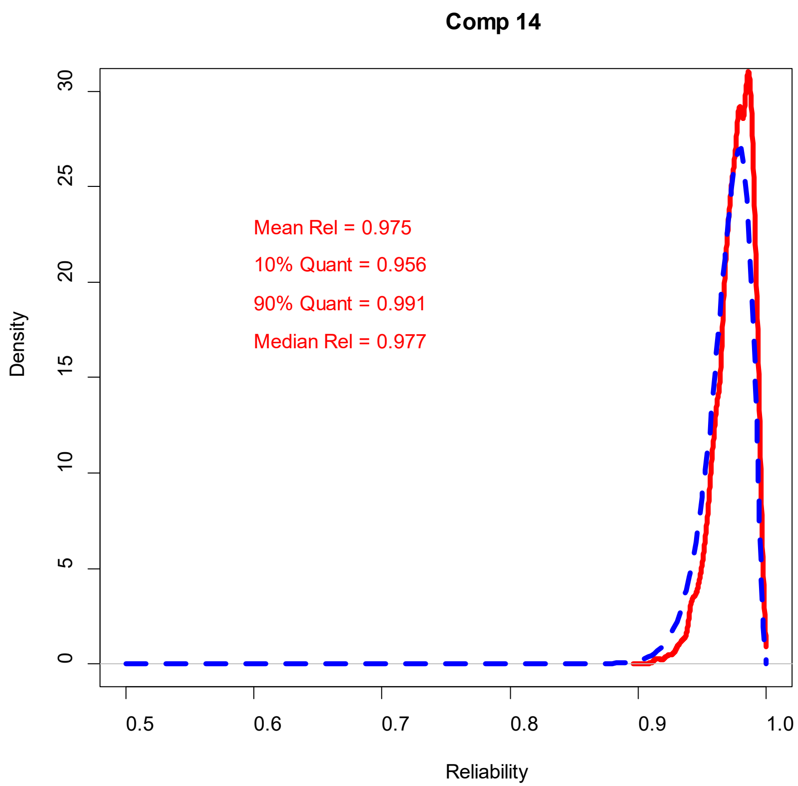
13 & 14





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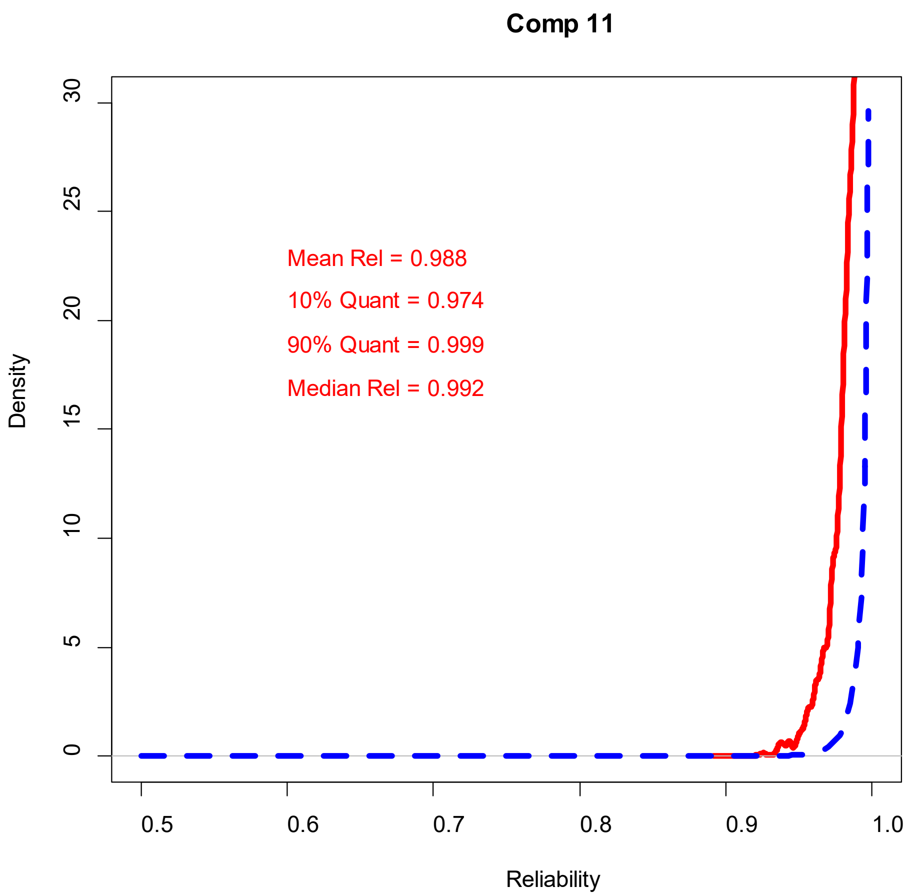

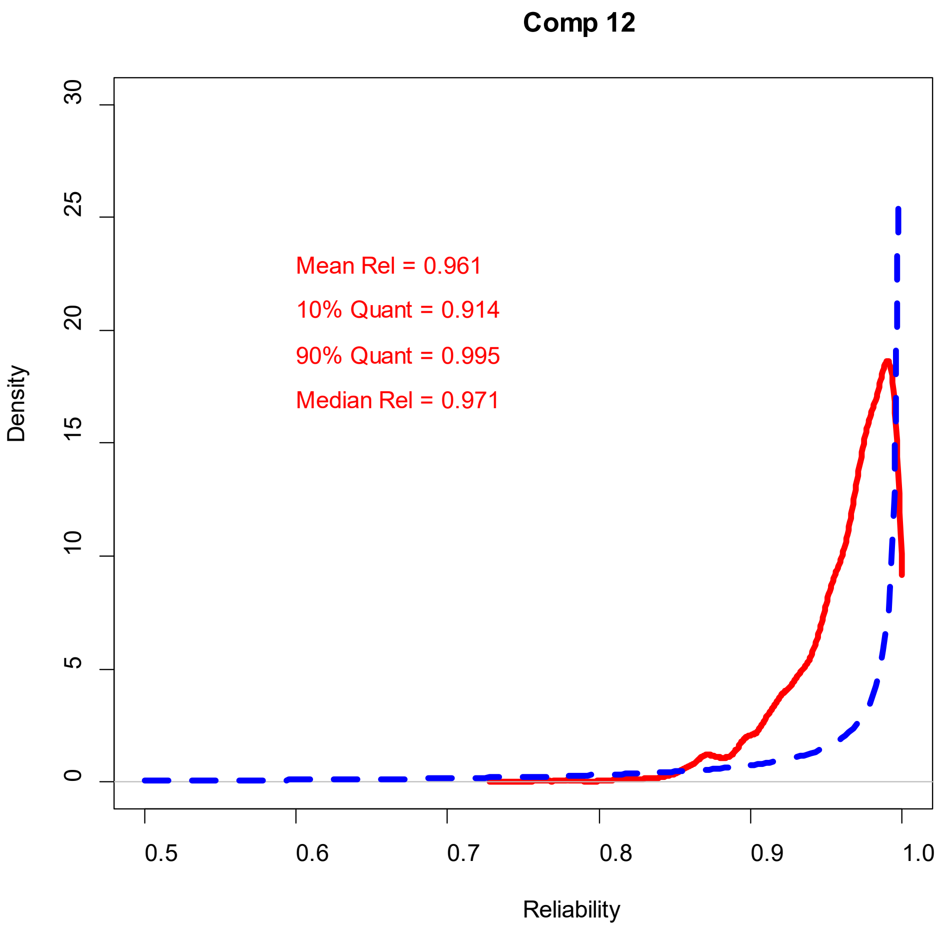

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Summary

- **Production Decision Represents the Largest Commitment of Resources with the Least System Level Information During Program Execution**
- **Clearly Identifying and Characterizing the Transition from DFR to RGT Provides Sound Basis for Evaluating RGP Progress Towards a Mature System for Use in Program Decisions**
- **A Bayesian Approach Allows Combining System Level Testing with Other Data and Subject Matter Inputs to Provide a More Accurate Evaluation of Current Reliability**
- **The Simple Bayesian Approach Presented Provides a Useful and Easy to Use Tool to Support Program Decisions and Allocation of Resources**