

# Petri Net Models of Adversarial Scenarios in Safety and Security

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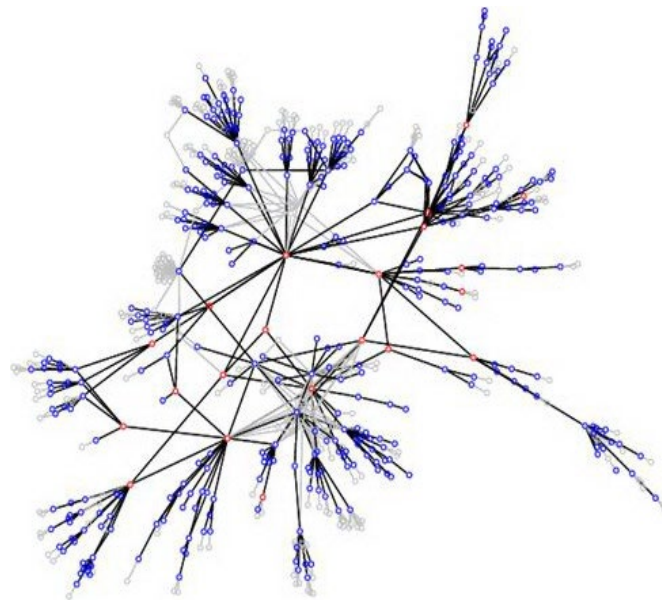
# Outline

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- Adversarial scenarios with concurrent autonomous/interactive actors
- Modeling methodologies
  - Markov and semi-Markov processes (flowgraphs)
  - Stochastic game theory
  - Event graphs
  - Bayesian networks
- Overview of Petri nets
  - General concepts, history
  - Generalized stochastic Petri nets
  - Use as a scenario elicitation tool
  - Use as a simulation tool
- Simulation of Petri nets
- Application example

# Adversarial systems and scenarios

- **System:** a group of entities that interact to function as a whole
  - We view systems as ***evolving, concurrent streams of events and actions***
- **Adversarial systems** are characterized by conflict between parties with opposing goals
  - Actions of adversaries are concurrent and interdependent
  - Outcomes are known only probabilistically
- **Scenario:** a postulated sequence or development of events
- We look at tools for
  - Eliciting adversarial scenarios from subject matter experts
  - Quantitative modeling of scenario outcomes



# Modeling methodologies

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- State-space (e.g., Markov) models
- Event graphs
- Bayesian networks
- Stochastic game theory
  - Adds evolutionary/learning behavior to game theory
- Stochastic programming, evolutionary design, . . .

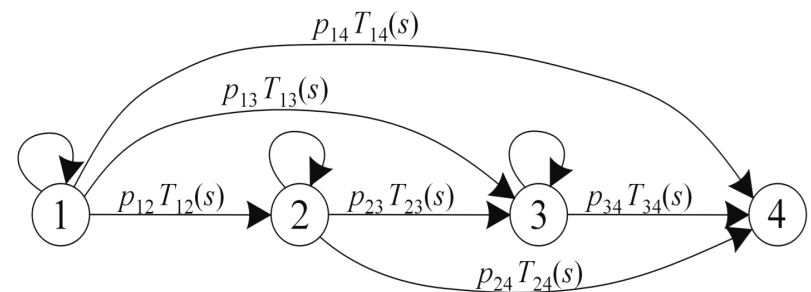
These methods (and others) assume sequential actions, serialized sample paths, or situations static in time.

“Adversaries” versus “defenders”  
implies multiple concurrently  
evolving streams of events



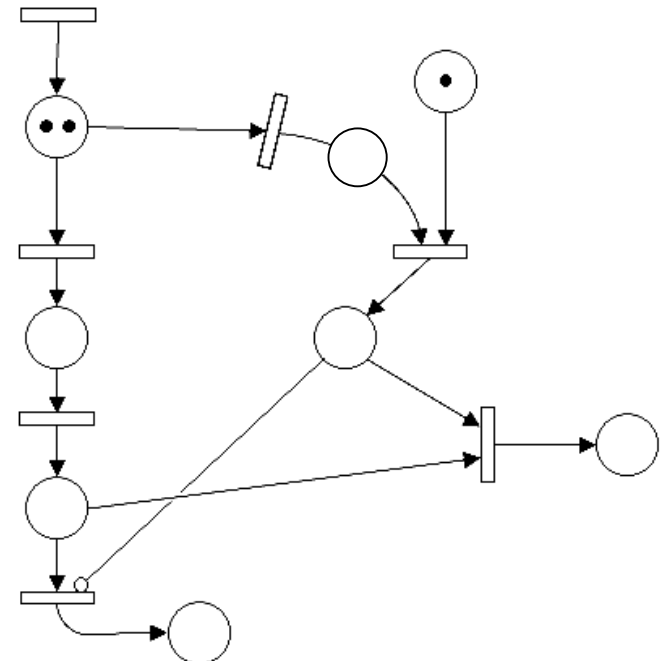
# Markovian stochastic networks

- Multistate models for predicting time to occurrence of an event (Huzurbazar 2005, Collins, Warr, and Huzurbazar 2013)
- Markov process – present state (not history) determines the future
  - Discrete-time, discrete-state Markov chain
  - Continuous time Markov chain (  $\Rightarrow$  exponential wait time distributions)
  - Semi-Markov process (arbitrary wait time distributions)
- Markov process extensions: lots of alternatives
  - Markov reward processes, Markov decision processes, hidden Markov, hidden semi-Markov models
  - $n$ th-order Markov processes (richer dependency between successive states)
- State-space models can handle arbitrarily complex scenarios, but ...
  - Cost is combinatorial explosion of states, loss of interpretability



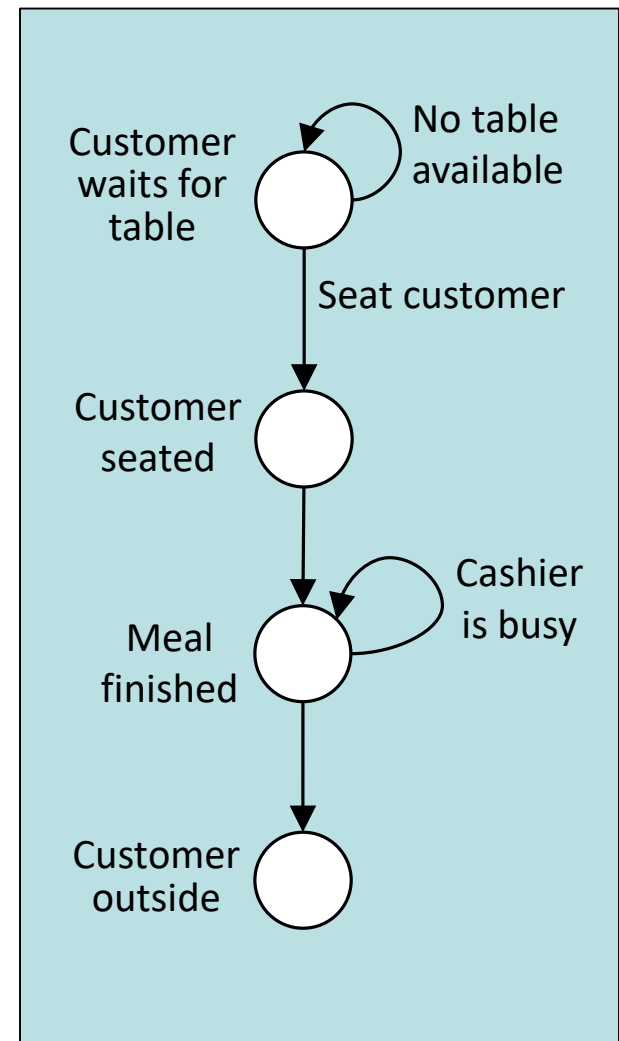
# Petri Nets

- Developed by Carl Petri for analysis of parallel computer architectures (Petri 1962)
  - Based on a strong mathematical foundation (Peterson 1977, Reisig 1982)
- Add multiple entities, concurrency to state transition diagrams
  - Places
  - Transitions
  - Tokens (represent actors, passive entities, or event triggers)
- Twofold purpose
  - A visual communication aid to elicit models of system behavior
  - A tool for developing quantitative simulation models



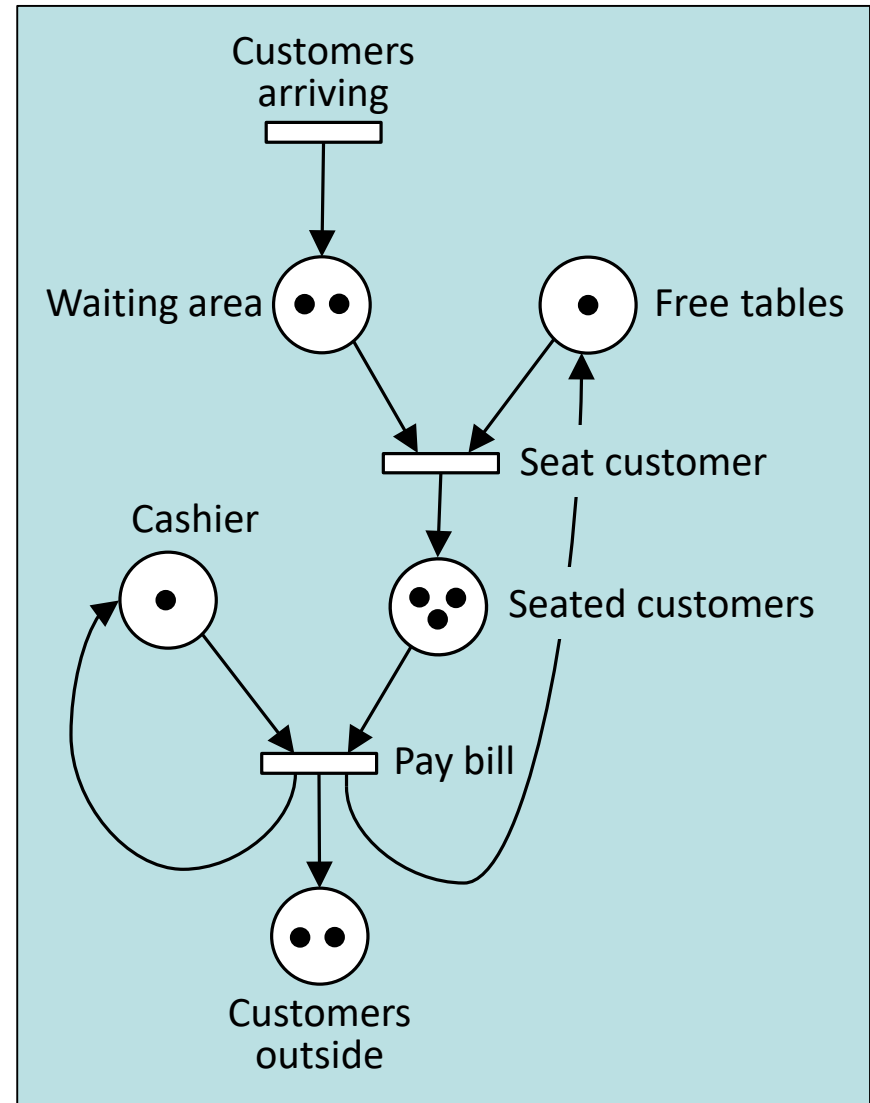
# Example: State transition model of restaurant service

- A state transition model represents the time history of a *single actor* (customer)
- Interaction between actors can only be modeled indirectly (e.g., “No table available,” “Cashier busy”)
- Multiple actors can only be incorporated by proliferating states (e.g., states for “Customer waits for one other,” “Customer waits for two others,” etc.)
- Deadlock or inability to reach a given state may occur due to factors that are not modeled by the state graph
- A more powerful representation is needed for modeling adversarial scenarios



# Petri net model of restaurant service

- *Tokens* represent concurrent actors or other entities occupying *places*
- *Transitions* fire when each input place has a token, and can generate multiple output tokens
- Multiple interacting actors can be represented in an obvious way
- Conflict and cooperation can easily be represented
- Mathematical formalism allows determination of deadlocks, reachable states, etc.





# Extensions to the basic Petri net model

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- Timed nets – deterministic transition times
- Stochastic nets
  - Explicit probabilities for non-deterministic transitions
  - Probabilistic transition times – Markov (exponential) or semi-Markov (arbitrary distribution)
- Logic extensions, e.g., inhibitory arcs
- Transitions that generate multiple tokens
- Hierarchical decomposition of nets

# Example scenario: Storage locker break-in

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Intruders penetrate a chain-link fence. With probability  $p = 0.9$ , a silent alarm is transmitted to a security company.

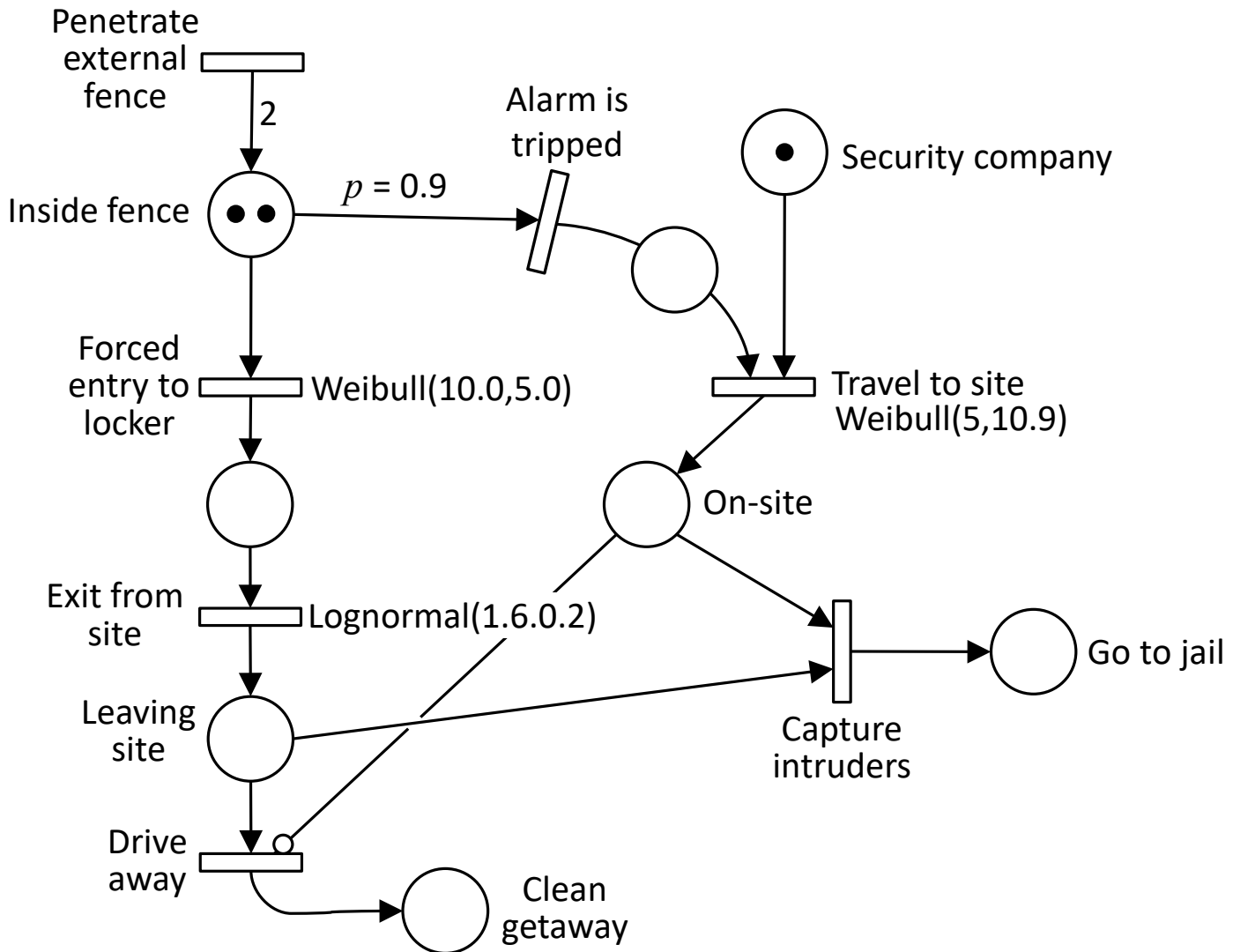
Patrol is dispatched from the security company; travel time varies depending on the location of the nearest patrol car that is able to respond (not on another call). Travel time  $T \sim \text{Weibull}(5, 10.9)$ .

Intruders forcibly open the storage locker. Time taken for this varies depending on the type of lock, etc.  $T \sim \text{Weibull}(10, 5)$ .

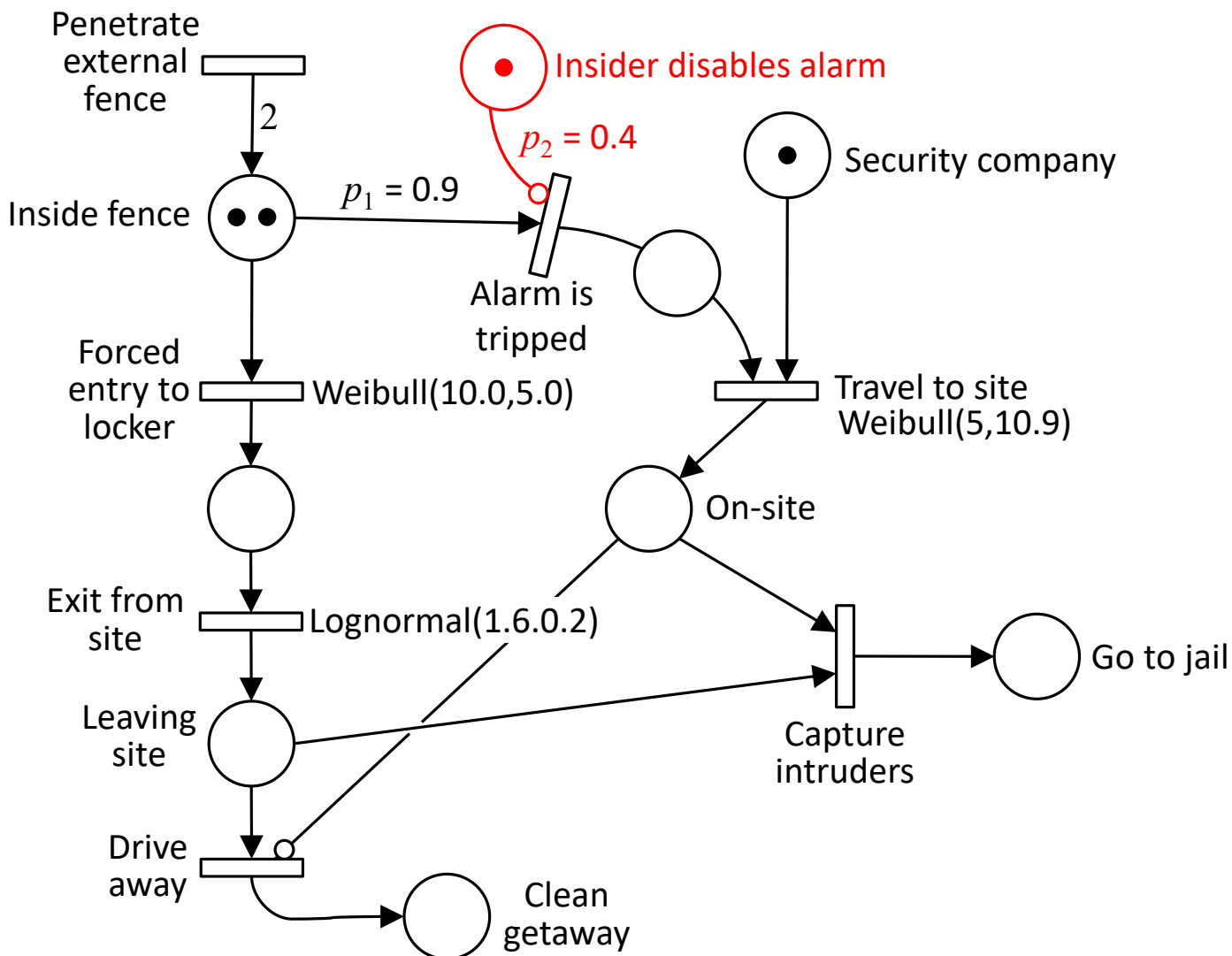
Intruders remove locker contents and exit. This is also a stochastic variable, distributed  $T \sim \text{Lognormal}(1.6, 0.2)$ .

Concurrently, if the alarm was tripped the security patrol has been traveling to the scene. If the patrol arrives before the intruders exit, they are captured. If not, they make a clean getaway.

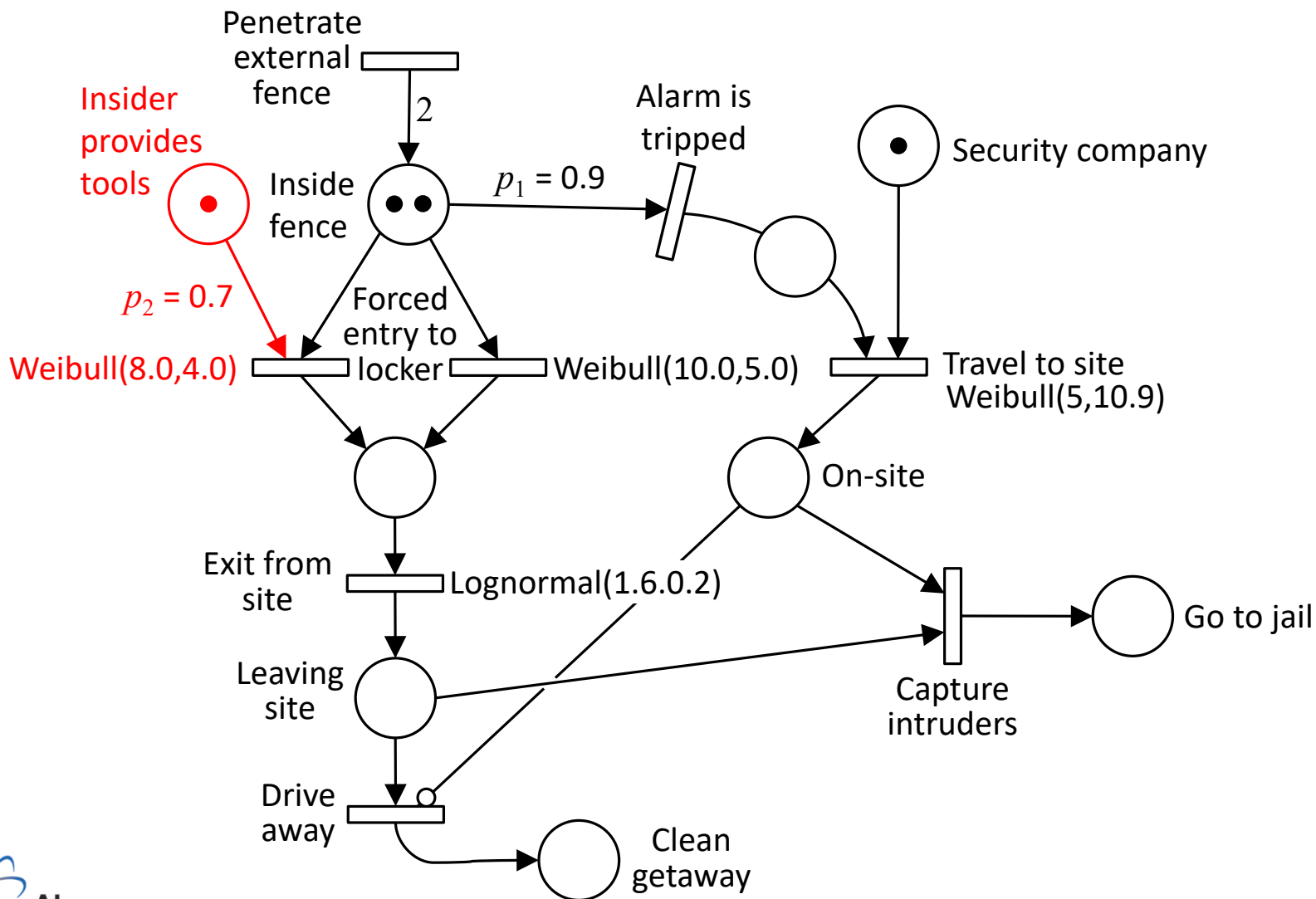
# Petri net model for storage locker break-in



# Storage locker break-in (insider threat case 1)



# Storage locker break-in (insider threat case 2)



# Implementation options for Petri net simulation

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- We are using the statistical programming language R
- Scenario-specific procedural code
  - Fast implementation for simple nets
  - Error-prone for complex nets, difficult to debug
  - Can't be extended to provide user-friendly net definition, graphical user interface
- Generalized object-oriented framework
  - More transparent: classes for Net, Place, Transition, etc.
  - Extensible to provide easy net definition, graphical user interface(s)
- Simulation allows sensitivity analysis, optimization over defensive countermeasures

# Example simulation output – storage locker break-in

Monte Carlo iterations: 10000

Elapsed time: 0.281 seconds

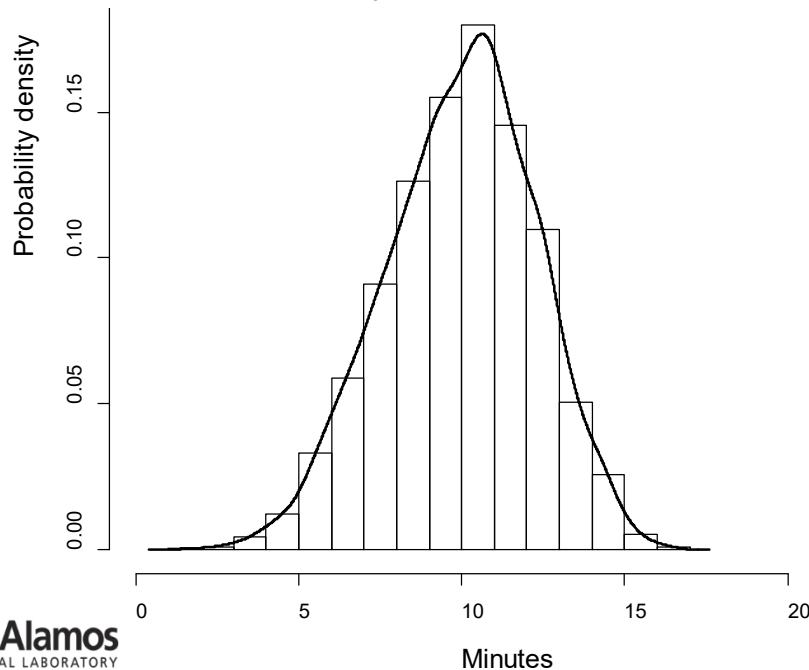
Probability of escape = 0.7479

Probability of security alarm = 0.5416

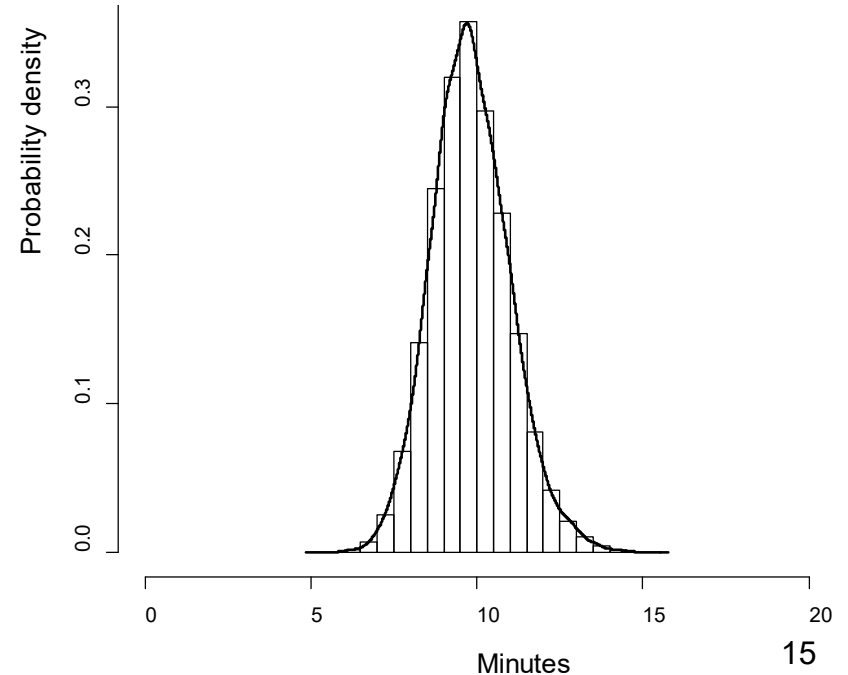
Mean travel time for the security company = 9.96 minutes

Mean exit time for the intruders = 9.82 minutes

Security arrival times



Intruder exit times



# Summary

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- Approaches to modeling adversarial scenarios with concurrent autonomous/interactive actors
  - Markov and semi-Markov processes (flowgraphs)
  - Stochastic game theory
  - Event graphs
  - Bayesian networks
- Petri nets overcome some deficiencies of other methods
  - Ability to model parallel, concurrent flows of events (e.g., attacker and defender actions)
  - Stochastic extensions allow statistical analysis (including Bayesian)
  - Can be used as a scenario elicitation tool, as well as for simulation
- Ongoing work
  - Analysis/representation of more complex scenarios
  - Object-oriented Petri net simulation framework
  - Optimization over defender actions and costs



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