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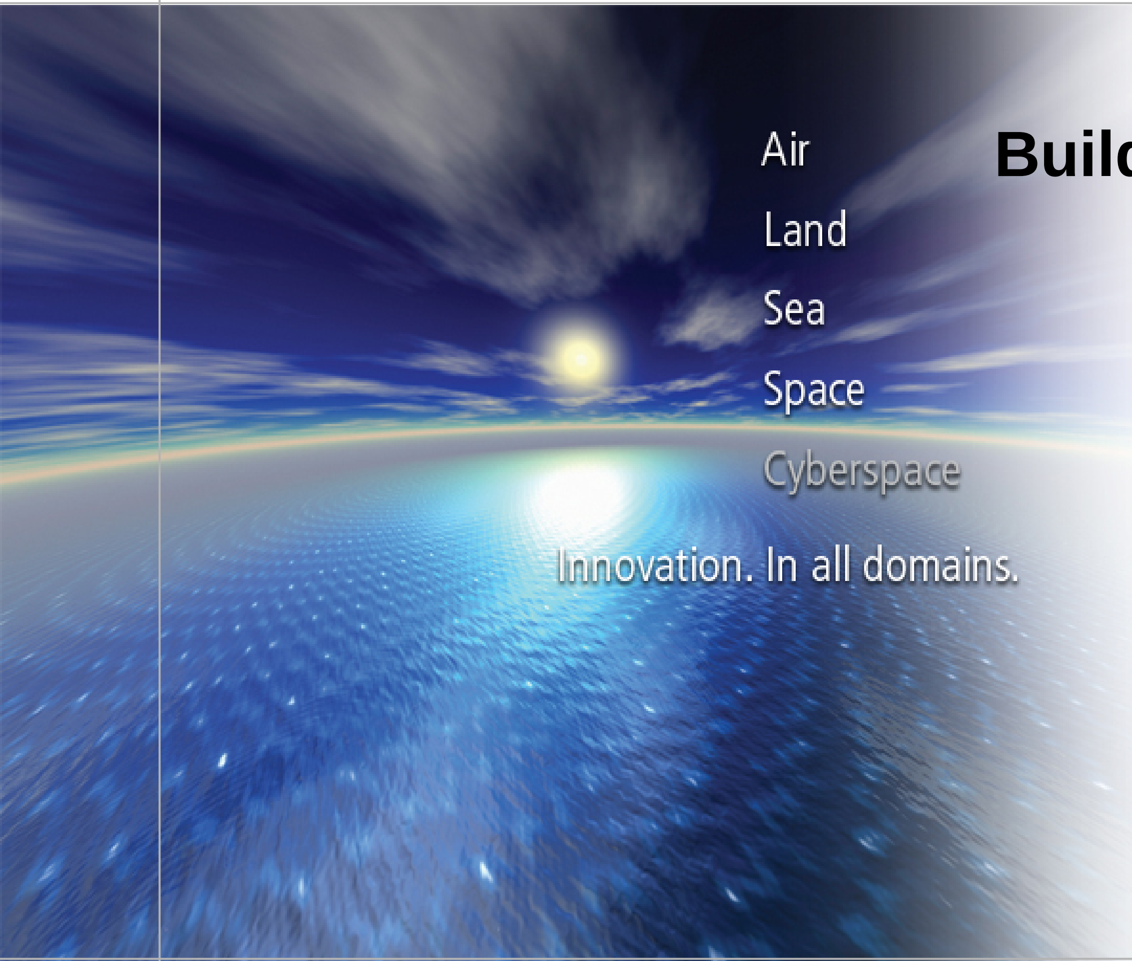

Building Requirements-Flow Models using Bayesian Networks and Designed Simulation Experiments

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Outline

- **Background & Motivation**
  - The power of probability models
  - Requirements-flow models for subsystem requirements
- **Basics of Bayesian Networks and the synergy between BNs and Design & Analysis of Simulation Experiments (DASE)**
  - Examples: Rain-Sprinkler-Grass & Chair-Backache BNs
  - DASE terminology in context of BNs
- **Example Requirements-Flow Model: Weapon Kill-Chain**
  - Improving performance vs. a prior, similar system
  - Using inference engine and DASE to verify solution
- **Summary & Conclusions**

Harnessing the power of probability

- **E.T. Jaynes** (*Probability Theory: The Logic of Science*, Cambridge, 2003)

  “The mathematical rules of probability theory are not merely rules for calculating frequencies of ‘random variables’; they are also the unique consistent rules for conducting inference (i.e. plausible reasoning).”

- **Definition of conditional probability:** $\Pr\left(A \mid B\right) = \frac{\Pr(A,B)}{\Pr(B)}$

  Numerator is the joint probability of (simple or compound) events A & B, and denominator is the marginal probability of “evidence event” B

- **Typical process for deriving subsystem requirements**
  - Ad-hoc quantitative & graphical methods are used to derive “flow-down” requirements
  - When compliance problems occur, issues arise when referring to artifacts of these methods (e.g., confusion between expressions of joint, conditional, and marginal probabilities)

Challenge: To create a logically consistent, probabilistic requirements-flow model

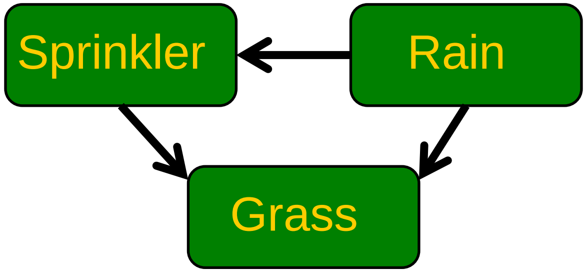
Example 1 (http://en.wikipedia.org/wiki/Bayesian_network)

Wet Grass, **G**; Sprinkler On **S**, & Raining, **R**

CPT$_S$ (1 parent)

R	S	
	F	T
F	0.6	0.4
T	0.99	0.01

causally directed acyclic graph (DAG)



CPT$_R$ (0 parents)

R	
F	T
0.8	0.2

Conditional Probability Tables (CPTs)

- Represent nodal events using discrete probability distributions
- Cells populated using data or best-guesses

CPT$_G$ (2 parents)

R	S	G	
		F	T
F	F	1	0
T	F	0.2	0.8
F	T	0.1	0.9
T	T	0.01	0.99

Using rules of probability (e.g., JPD factorization, Law of Total Probability, & Bayes' Rule), a BN model is sufficient for computing correct probabilistic answers to specific questions.

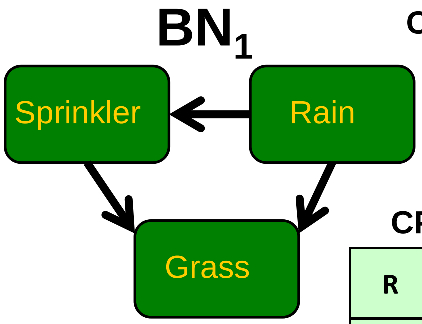
Example 1 (G, S, R), Cont.:
Computation for a specific query

Q1: What is the probability that it's raining, given that the grass is wet?

Start with conditional probability formula, and sum over "nuisance" variable(s) X:

CPT_S (1 parent)

R	S	
	F	T
F	0.6	0.4
T	0.99	0.01



CPT_R (0 parents)

R	
F	T
0.8	0.2

CPT_R (2 parents)

R	S	G	
		F	T
F	F	1	0
T	F	0.2	0.8
F	T	0.1	0.9
T	T	0.01	0.99

$$\Pr(R|G) = \frac{\Pr(G,R)}{\Pr(G)} = \frac{\displaystyle\sum_{X\in\{S,!S\}} \Pr(G,X,R) \quad \leftarrow 2^1 = 2 \text{ products}}{\displaystyle\sum_{\substack{X_1\in\{S,!S\}\\ X_2\in\{R,!R\}}} \Pr(G,X_1,X_2) \quad \leftarrow 2^2 = 4 \text{ products}}$$

!X is the complement of X, i.e. 1 - X

$$= \frac{\Pr(G|S,R)\Pr(S|R)\Pr(R) + \Pr(G|!S,R)\Pr(!S|R)\Pr(R)}{\Pr(G|S,R)\Pr(S|R)\Pr(R) + \Pr(G|S,!R)\Pr(S|!R)\Pr(!R) + \Pr(G|!S,R)\Pr(!S|R)\Pr(R) + \Pr(G|!S,!R)\Pr(!S|!R)\Pr(!R)}$$

$$= \frac{(0.99\times0.01\times0.2)+(0.8\times0.99\times0.2)}{(0.99\times0.01\times0.2)+(0.9\times0.4\times0.8)+(0.8\times0.99\times0.2)+(0\times0.6\times0.8)} = \frac{0.16038}{0.44838} = 0.358$$

∴ A1: The probability that it's raining, given wet grass, is ~36%.

Example 2*: Aching Back, **A**; Injury, **B**; Other Worker(s), **W**; Sports Injury, **S**; & Bad Chair, **C**

*From Ben-Gal, "Bayesian Networks," www.eng.tau.ac.il/~bengal/BN.pdf

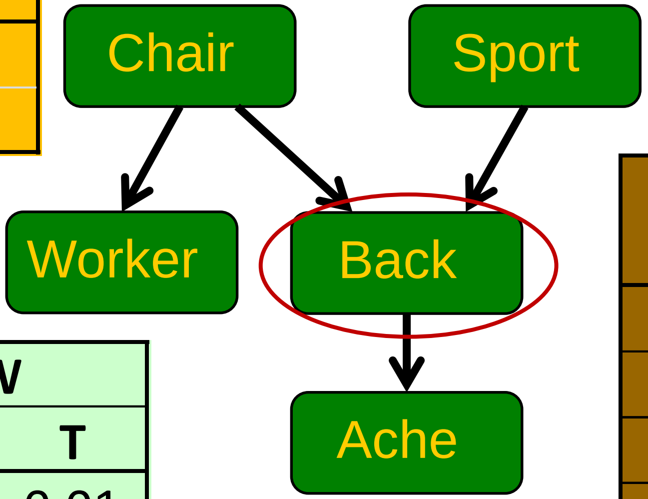
BN$_2$

CPT$_C$ (0 parents)

C	
F	T
0.2	0.8

CPT$_S$ (0 parents)

S	
F	T
0.98	0.02



C	W	
	F	T
F	0.99	0.01
T	0.1	0.9

S	C	B	
		F	T
F	F	0.99	0.01
T	F	0.1	0.9
F	T	0.8	0.2
T	T	0.1	0.9

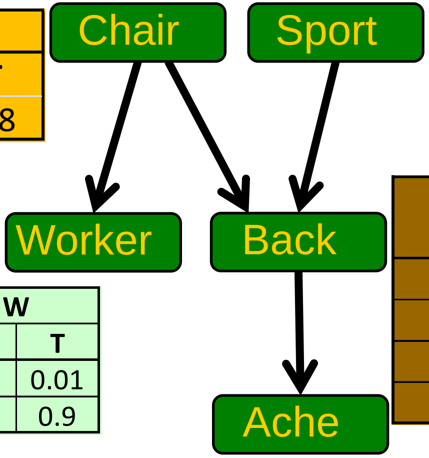
B	A	
	F	T
F	0.9	0.1
T	0.3	0.7

Q2: What is the probability that Bob's chair is bad, given that his back aches?

Example 2 (A,B,W,S,C), cont.: Computation for specific question

Q2: What is the probability that Bob's chair is bad, given that his back aches?

C	
F	T
0.2	0.8



S	
F	T
0.98	0.02

S	C	B	
		F	T
F	F	0.99	0.01
T	F	0.1	0.9
F	T	0.8	0.2
T	T	0.1	0.9

C	W	
	F	T
F	0.99	0.01
T	0.1	0.9

B	A	
	F	T
F	0.9	0.1
T	0.3	0.7

$$\Pr(C\,|\,A)=\frac{\Pr(A,C)}{\Pr(A)}=\frac{\displaystyle\sum_{\substack{X_1\in\{B,!B\}\\ X_2\in\{W,!W\}\\ X_3\in\{S,!S\}}}\Pr(A,X_1,X_2,X_3,C)}{\displaystyle\sum_{\substack{X_1\in\{B,!B\}\\ X_2\in\{W,!W\}\\ X_3\in\{S,!S\}\\ X_4\in\{C,!C\}}}\Pr(A,X_1,X_2,X_3,X_4)}$$

$\leftarrow 2^3 = 8$ products

$\leftarrow 2^4 = 16$ products

Omit "Pr" to save space

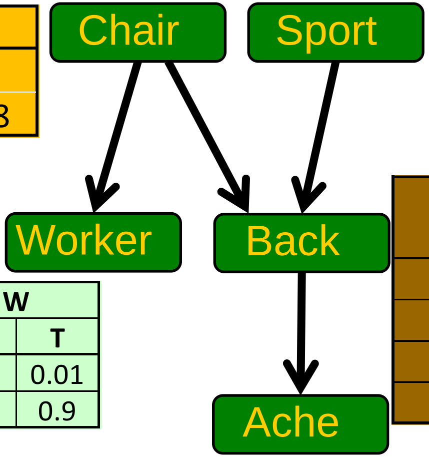
$$=\frac{\begin{array}{l}(A|B)(B|S,C)(W|C)(S)(C)+(A|B)(B|!S,C)(W|C)(!S)(C)\\ +(A|B)(B|S,C)(!W|C)(S)(C)+(A|B)(B|,C)(!W|C)(!S)(C)\\ +(A|!B)(!B|S,C)(W|C)(S)(C)+(A|!B)(!B|C)(W|C)(!S)(C)\\ +(A|!B)(!B|S,C)(!W|C)(S)(C)+(A|!B)(!B|C)(!W|C)(!S)(C)\end{array}}{\begin{array}{l}(A|B)(B|S,C)(W|C)(S)(C)+(A|B)(B|S,!C)(W|!C)(S)(!C)\\ +(A|B)(B|!S,C)(W|C)(!S)(C)+(A|B)(B|!S,!C)(W|!C)(!S)(!C)\\ +(A|B)(B|S,C)(!W|C)(S)(C)+(A|B)(B|S,!C)(!W|!C)(S)(!C)\\ +(A|B)(B|!S,C)(!W|C)(!S)(C)+(A|B)(B|!S,!C)(!W|!C)(!S)(!C)\\ +(A|!B)(!B|S,C)(W|C)(S)(C)+(A|!B)(!B|S,!C)(W|!C)(S)(!C)\\ +(A|!B)(!B|!S,C)(W|C)(!S)(C)+(A|!B)(!B|!S,!C)(W|!C)(!S)(!C)\\ +(A|!B)(!B|S,C)(!W|C)(S)(C)+(A|!B)(!B|S,!C)(!W|!C)(S)(!C)\\ +(A|!B)(!B|!S,C)(!W|C)(!S)(C)+(A|!B)(!B|!S,!C)(!W|!C)(!S)(!C)\end{array}}=\frac{Num}{Den}$$

Example 2, cont.: Numerator Computation

$$\Pr(C\,|\,A)=\frac{\Pr(A,C)}{\Pr(A)}=\frac{\displaystyle\sum_{\substack{X_1\in\{B,!B\}\\X_2\in\{W,!W\}\\X_3\in\{S,!S\}}}\Pr(A,X_1,X_2,X_3,C)}{\displaystyle\sum_{\substack{X_1\in\{B,!B\}\\X_2\in\{W,!W\}\\X_3\in\{S,!S\}\\X_4\in\{C,!C\}}}\Pr(A,X_1,X_2,X_3,X_4)}$$

← 2^3 = 8 products

← 2^4 = 16 products



C	
F	T
0.2	0.8

S	
F	T
0.98	0.02

C	W	
	F	T
F	0.99	0.01
T	0.1	0.9

S	C	B	
		F	T
F	F	0.99	0.01
T	F	0.1	0.9
F	T	0.8	0.2
T	T	0.1	0.9

B	A	
	F	T
F	0.9	0.1
T	0.3	0.7

$$\left(\begin{array}{l}
(A|B)(B|S,C)(W|C)(S)(C)+(A|B)(B|!S,C)(W|C)(!S)(C)\\
\quad 0.7\times 0.9\times 0.9\times 0.02\times 0.8+0.7\times 0.2\times 0.9\times 0.98\times 0.8\\
+(A|B)(B|S,C)(!W|C)(S)(C)+(A|B)(B|!S,C)(!W|C)(!S)(C)\\
\quad 0.7\times 0.9\times 0.1\times 0.02\times 0.8+0.7\times 0.2\times 0.1\times 0.98\times 0.8\\
+(A|!B)(!B|S,C)(W|C)(S)(C)+(A|!B)(!B|!S,C)(W|C)(!S)(C)\\
\quad 0.1\times 0.1\times 0.9\times 0.02\times 0.8+0.1\times 0.8\times 0.9\times 0.98\times 0.8\\
+(A|!B)(!B|S,C)(!W|C)(S)(C)+(A|!B)(!B|!S,C)(!W|C)(!S)(C)\\
\quad 0.1\times 0.1\times 0.1\times 0.02\times 0.8+0.1\times 0.8\times 0.1\times 0.98\times 0.8
\end{array}\right)=0.1827$$

Example 2, cont.: Denominator computation

As the number of nodes k and the number of states per node m increase, the computational complexity for a general joint probability distribution grows as $O(m^k)$; with BNs, it grows more slowly, as square of the maximum number of parents per node.

$$\left(\begin{array}{l}
(A|B)(B|S,C)(W|C)(S)(C) + (A|B)(B|S,!C)(W|!C)(S)(!C) \\
0.7 \times 0.9 \times 0.9 \times 0.02 \times 0.8 + 0.7 \times 0.9 \times 0.01 \times 0.02 \times 0.2 \\
+(A|B)(B|!S,C)(W|C)(!S)(C) + (A|B)(B|!S,!C)(W|!C)(!S)(!C) \\
0.7 \times 0.2 \times 0.9 \times 0.98 \times 0.8 + 0.7 \times 0.01 \times 0.01 \times 0.98 \times 0.2 \\
+(A|B)(B|S,C)(!W|C)(S)(C) + (A|B)(B|S,!C)(!W|!C)(S)(!C) \\
0.7 \times 0.9 \times 0.1 \times 0.02 \times 0.8 + 0.7 \times 0.9 \times 0.99 \times 0.02 \times 0.2 \\
+(A|B)(B|!S,C)(!W|C)(!S)(C) + (A|B)(B|!S,!C)(!W|!C)(!S)(!C) \\
0.7 \times 0.2 \times 0.1 \times 0.98 \times 0.8 + 0.7 \times 0.01 \times 0.99 \times 0.98 \times 0.2 \\
+(A|!B)(!B|S,C)(W|C)(S)(C) + (A|!B)(!B|S,!C)(W|!C)(S)(!C) \\
0.1 \times 0.1 \times 0.9 \times 0.02 \times 0.8 + 0.1 \times 0.1 \times 0.01 \times 0.02 \times 0.2 \\
+(A|!B)(!B|!S,C)(W|C)(!S)(C) + (A|!B)(!B|!S,!C)(W|!C)(!S)(!C) \\
0.1 \times 0.8 \times 0.9 \times 0.98 \times 0.8 + 0.1 \times 0.99 \times 0.01 \times 0.98 \times 0.2 \\
+(A|!B)(!B|S,C)(!W|C)(S)(C) + (A|!B)(!B|S,!C)(!W|!C)(S)(!C) \\
0.1 \times 0.1 \times 0.1 \times 0.02 \times 0.8 + 0.1 \times 0.1 \times 0.99 \times 0.02 \times 0.2 \\
+(A|!B)(!B|!S,C)(!W|C)(!S)(C) + (A|!B)(!B|!S,!C)(!W|!C)(!S)(!C) \\
0.1 \times 0.8 \times 0.1 \times 0.98 \times 0.8 + 0.1 \times 0.99 \times 0.99 \times 0.98 \times 0.2
\end{array}\right) = 0.2061$$

Example 2 (A,B,W,C,S), cont.
Computation for specific question

A2: The probability that Bob's chair is bad, given that his back aches is

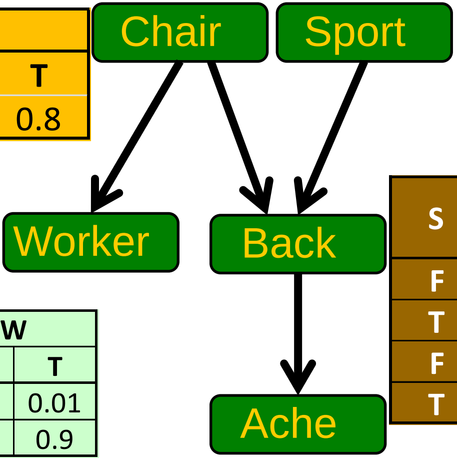
$$\Pr\left(C \mid A\right) = \frac{\Pr(A,C)}{\Pr(A)} = \frac{\displaystyle\sum_{\substack{X_1\in\{B,!B\}\\ X_2\in\{W,!W\}\\ X_3\in\{S,!S\}}} \Pr\left(A, X_1, X_2, X_3, C\right)}{\displaystyle\sum_{\substack{X_1\in\{B,!B\}\\ X_2\in\{W,!W\}\\ X_3\in\{S,!S\}\\ X_4\in\{C,!C\}}} \Pr\left(A, X_1, X_2, X_3, X_4\right)}$$

← $2^3 = 8$ products

← $2^4 = 16$ products

$$= \frac{0.1827}{0.2061} = 0.8867$$

C	
F	T
0.2	0.8



S	
F	T
0.98	0.02

S	C	B	
		F	T
F	F	0.99	0.01
T	F	0.1	0.9
F	T	0.8	0.2
T	T	0.1	0.9

C	W	
	F	T
F	0.99	0.01
T	0.1	0.9

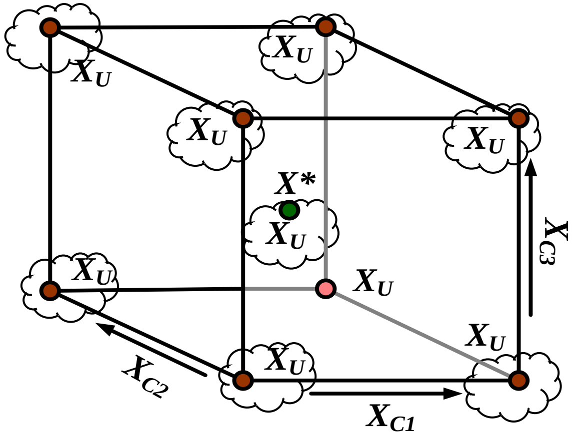
B	A	
	F	T
F	0.9	0.1
T	0.3	0.7

∴ A2: The probability that Bob's chair is bad, given that his back aches, is ~89%.

As demonstrated on previous slides, we definitely need software (*inference engine*) to do these computations correctly, given a valid model (BN + nodal state definitions + CPTs). Examples: Kevin Murphy's Bayes Net Toolbox: http://code.google.com/p/bnt/ & UCLA's SamIam: http://reasoning.cs.ucla.edu/samiam/

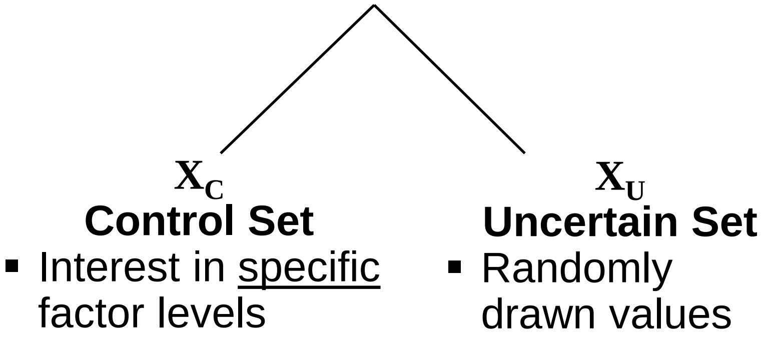
Synergy of DASE, Design & Analysis of Simulation Experiments, with BNs

The Factor Hypercube*



At each chosen point in the X_C space, a set of random draws from factor set X_U is made.

Factor-Set Assignment



X_C

Control Set

- Interest in specific factor levels

X_U

Uncertain Set

- Randomly drawn values

*The most common DASE experimental design is space-filling, i.e. Latin Hypercube Sampling, to obtain response *summary statistics,* which—in the case of BNs—are used to populate CPT entries.

DASE is used in the 1st and 3rd stages of requirements-flow modeling:

1) Populate BN's CPT entries with baseline sample proportion estimates
2) "Judiciously" modify baseline CPT entries, weighing cost/benefit options
3) Verify that new design satisfies the BN's modified CPT entries and system performance requirements, which are stated as marginal probabilities

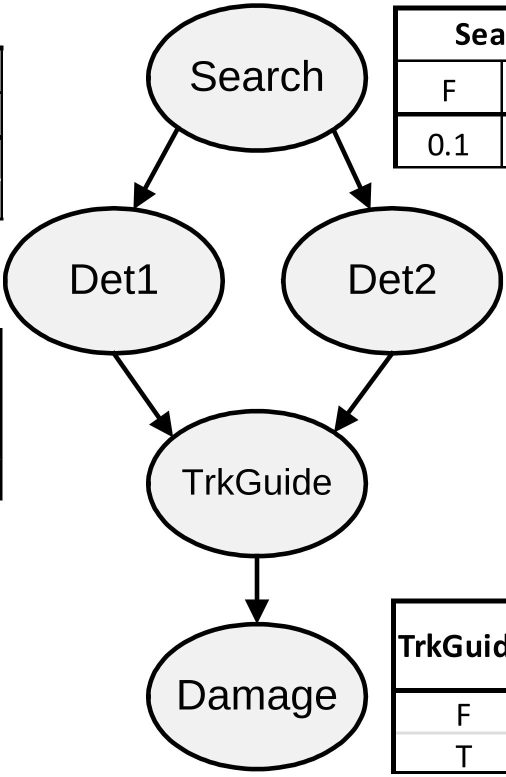
Example 3*
Weapon kill-chain model for anti-aircraft missile

*Simplified from Ball's example in *The Fundamentals of Aircraft Combat Survivability Analysis & Design* (2^{nd} ed., 2003)

Search
F	T
0.1	0.9

Search	Det1	
	F	T
F	1	0
T	0.1	0.9

Search	Det2	
	F	T
F	1	0
T	0.4	**0.6**



Det1	Det2	TrkGuide	
		F	T
F	F	0.55	0.45
F	T	0.12	0.88
T	F	0.4	0.6
T	T	0.1	0.9

TrkGuide	Damage		
	None	Disable	Kill
F	1	0	0
T	0.03	0.02	0.95

Observe the higher conditional potential of Sensor2 vs. Sensor1.[2]

Exploring potential of increasing this sensor's performance in order to increase P_K[1] value from baseline system's *0.70* to new system's *0.77*

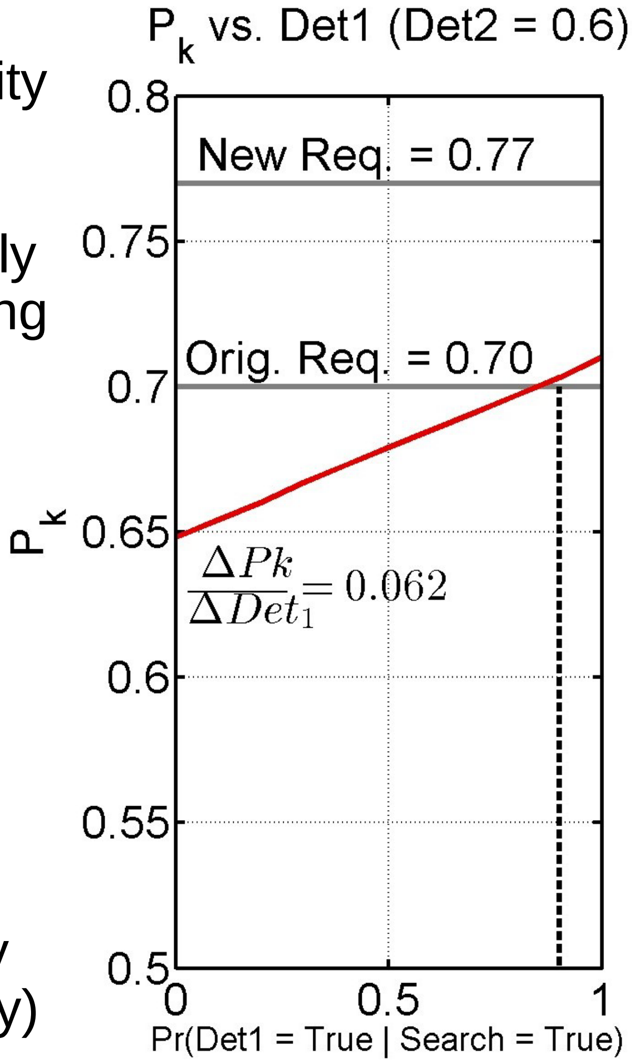
[1] Common error: to confuse a conditional probability like $Pr(Damage=Kill \mid TrkGuide = True)$, with a marginal probability like P_K

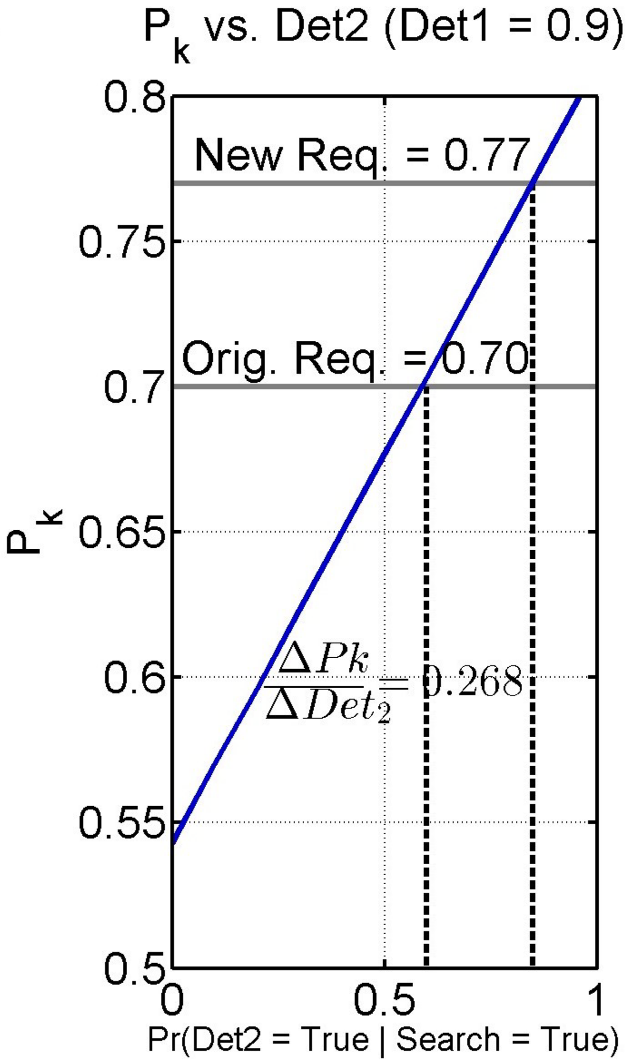
[2] We're ignoring potential for improving both sensors' performance simultaneously—along with many more complex, allowable queries of this BN!

Sensitivity Analysis

Weapon kill-chain model for anti-aircraft missile

- Simple, 1-factor sensitivity queries for each sensor quantify the higher potential for incrementally improving P_K by improving Sensor2's performance than Sensor1's
- In addition, Sensor1's performance is already closer to 1; incremental improvements are likely more difficult to obtain
- Therefore, the better decision is to improve Sensor2 (and then verify with another DASE study)





Many, more-complex queries can be explored using the same BN.

Summary

- **Combined with DASE, Bayesian networks (BNs) provide a way to allocate & evaluate subsystem requirements in a quantitative, comprehensive manner**
  - A rigorous probability model enables immediate evaluation of “what-if” queries when considering subsystem improvements
  - A requirements-flow model enables immediate sensitivity analysis and upper-bound estimates on the likely achievable gains of a proposed improvement
  - BNs provide a natural framework for integrating results from multiple DASE studies to mitigate suboptimization
- **Many tools exist for DASE and BN development**

The challenge: To educate engineers in the promise of BNs + DASE and the use of available tools.