Multi-Criteria Decision Analysis on Aircraft Stringer Selection

Dr. Shuguang Song
The Boeing Company

Joint work with You Ren and Paul D. Sampson
University of Washington

October 22, 2014

Abstract

Multi-Criteria Decision Analysis (MCDA) problems often involve multiple Decision Makers(DMs). In this paper, we present several decision analysis algorithms, considering both subjective and objective decision criteria with different strategies to account for uncertainty. We address the uncertainty and availability of weights for decision criteria, and develop probability scoring for the criteria. We demonstrate an application of our method with a case study concerning aircraft stringer decisions.

Introduction

- **Categorize the decision criteria into subjective decision criteria and objective decision criteria.**
  - To model the uncertainty of subjective criteria, we sample the data from a Bayesian posterior distribution.
  - To model the uncertainty of objective criteria, we sample the data from appropriate probability distribution or empirical distribution.
- **Study the uncertainty in weights from multiple DMs by treating weight as a subjective criterion.**
- **Develop a probability score with embedded sampling procedure to measure the probability that one alternative outperforms another.**
- **Implemented five MCDA algorithms**
  - 1-stage sampling + normalization table
  - 2-stage sampling + normalization table
  - 1-stage sampling + interval hull linear score
  - 2-stage sampling + interval hull linear score
  - probability score (related to the pairwise winning index)

Bayesian 2-Stage Sampling Procedure

- **Let X_i be the value for a subjective decision criterion from i-th DM, i =1, …, d.**
- **$X_i | \vec{p} \sim \text{Multinominal}(\vec{p})$, i= 1, …, d**
- **Prior distribution: $\vec{p} \sim \text{Dirichlet}(\vec{\alpha}=1,\ldots,1)$**
- **Posterior distribution: $\vec{p} | X_1, \ldots, X_d \sim \text{Dirichlet}\ (\vec{\gamma}=\vec{\alpha}+\vec{\beta})$, where $\beta_r = \sum_{s=1}^{d} \mathbf{1}_{[Xs=r]}$.**

- **2-Stage Bayesian sampling: for t from 1 to M,**
  - Step 1: Sample one $\vec{p}_t$ from its posterior distribution $\vec{p} \sim \text{Dirichlet}\ (\vec{\gamma})$.
  - Step 2: Sample one data value X_t from its distribution $X_t \sim \text{Multinomial}(\vec{p}_t)$.

Bayesian 1-Stage Sampling Procedure

- **The optimal parameter estimate I the one that minimize the quadratic loss function, which is the mean of the posterior distribution.**
- $\widehat{\vec{p}} = \frac{\vec{\gamma}}{1^T \vec{\gamma}}$
- **Algorithm: obtain posterior mean $\widehat{\vec{p}} = \frac{\vec{\gamma}}{1^T \vec{\gamma}}$. For t from 1 to M, sample one data value X_t from its distribution $X_t \sim$ Multinomial($\widehat{\vec{p}}$).**
- **Both 1-stage sampling procedure and 2-stage sampling procedure can be used to continuous subjective decision criterion by discretization.**

Probability Score

- **Let X_1 denote the criterion variable for alternative 1, and X_2 denote the criteria variable for alternative 2.**

- **If the higher the criterion value, the better the alternative**
  - **probability score of X_1 is $\Pr(X_1 > X_2)$**
  - **probability score of X_2 is $\Pr(X_2 > X_1)$**

- **If the lower the criterion value, the better the alternative**
  - **probability score of X_1 is $\Pr(X_2 > X_1)$**
  - **probability score of X_2 is $\Pr(X_2 < X_1)$**

- **If categorical variable, take $\Pr(X_1 > X_2) + 0.5 \Pr(X_1 = X_2)$**

- **Rescale original probability score from [0,1] to [-1, 1].**

Probability Score: objective criteria

- **Subjective criterion: for sample t from 1 to M,**
  - Step 1: Sample one $\vec{p}_t^1$ for Alternative 1, $\vec{p}_t^2$ for Alternative 2 from their posterior distributions.
  - $X_t^1 \sim$ Multinomial($\vec{p}_t^1$), $X_t^2 \sim$ Multinomial($\vec{p}_t^2$). $S_{12}=\Pr(X_t^1 > X_t^2)$ or $\Pr(X_t^1 < X_t^2)$.
  - Matrix of pairwise comparison

$$\begin{bmatrix} 0 & s_{12} & \cdots & s_{1m} \\ \vdots & 0 & \ddots & \vdots \\ s_{m1} & s_{m2} & \cdots & 0 \end{bmatrix}$$

  - Obtain probability score vector ($\bar{s}_1$,..., $\bar{s}_m$) for each alternative on one subjective criterion under sample t. Here, $\bar{s}_k = \sum_{l=1}^{m} \frac{s_{kl}}{m-1}$, which represents the average probability of alternative k outperforming the others.

- **Objective criterion:**
  - bootstrap observed values to get another sample, calculate $\Pr(X_t^1 > X_t^2)$
  - If the sample size is large, no need of bootstrapping.

Normalization Table and Interval Hull

- **Normalization table**
  - list the value range of each criterion and its associated score
  - provided by DMs.

- **Interval Hull Linear Mapping Method**
  - The interval hull for criterion j is the smallest interval that contains 95% confidence intervals of all M alternatives on criterion j.
  - The two end points of the interval hull are mapped to the least and the most preferable values in the utility function, i.e., 0 and 1.
  - The utility or mapping function is then assumed to be linear between the two end points.
  - See Tervonen et al. (2005)

An Illustrated Example: Aircraft Stringer Selection

CycleTimeAP.xls [Compatibility Mode]

	A	B	C	D
1	ID	Stringer I	Stringer II	Stringer III
2	1	560	396	255
3	2	654	379	281
4	3	595	442	246
5	4	587	364	218
6	5	637	416	272
7	6	485	344	256
8	7	558	394	265
9	8	552	480	244

Sheet1 Sheet2 Sheet3

LaborhoursAP.xlsx

	A	B	C	D
1	ID	Stringer I	Stringer II	Stringer III
2	1	2257	1858	1781
3	2	2351	1912	1504
4	3	2326	1770	1555
5	4	2220	2018	1630
6	5	2552	1800	1709
7	6	2232	2009	1864
8	7	2316	1949	1411
9	8	2390	1704	1439

Sheet1 Sheet2 Sheet3

cost.xls [Compatibility Mode]

	B	C	D
1	Stringer I	Stringer II	Stringer III
2	109	91	88

Sheet1 Sheet2 Sheet3

mappingTable_withDirectionOnly.xlsx

	A	B
1	Criteria	direction
2	cost	-1
3	reworkRate	-1
4	LaborhoursAP	-1
5	CycleTimeAP	-1
6	TRL	1

Sheet1 Sheet2 Shee

reworkRate.xls [Compatibility Mode]

	A	B	C	D
1	Expert ID	Stringer I	Stringer II	Stringer III
2	1	5%	15%	25%
3	2	8%	13%	20%
4	3	10%	13%	18%
5	4	10%	15%	15%
6	5	9%	10%	10%
7	6	13%	8%	18%
8	7	7%	16%	16%
9	8	7%	12%	14%
10	9	8%	15%	20%
11	10	14%	16%	25%

Sheet1 Sheet2 Sheet3

TRL.csv

	A	B	C	D
1	Expert ID	Stringer I	Stringer II	Stringer III
2	1	10	8	
3	2	10	9	6
4	3	9	10	5
5	4	8	8	9
6	5	10	8	7
7	6	8	7	7
8	7	9	6	8
9	8	7	7	8

TRL

weight.xls [Compatibility M...

	A	B	C
1	Decision Factor	expert1	expert2
2	cost	5	5
3	TRL	4	3
4	reworkRate	3	4
5	LaborhoursAP	2	2
6	CycleTimeAP	1	2

Sheet1 Sheet2

Aircraft Stringer Selection: Weight of Decision Criteria

Decision Factor	Weight from DM1	Weight from DM2
Cost	5	5
Cycle time per airplane	1	2
Labor hours per airplane	2	2
Rework rate	3	4
TRL	4	3

Normalization Table for Stringer Study

Score	10	9	8	7	6	5	4	3	2	1
cost	0-50	50-80	80-90	90-100	100-110	110-120	120-130	130-140	140-150	150-160
TRL	10	9	8	7	6	5	4	3	2	1
rework rate	0%-10%	10%-20%	20%-30%	30%-40%	40%-50%	50%-60%	60%-70%	70%-80%	80%-90%	90%-100%
labor hours	0.5K-1.0K	1.0K-1.5K	1.5K-2.0K	2.0K-2.5K	2.5K-3.0K	3.0K-3.5K	3.5K-4.0K	4.0K-4.5K	4.5K-5.0K	5.0K-5.5K
cycle time	100-200-	200-300-	300-400-	400-500-	500-600-	600-700-	700-800-	800-900-	900-1K-	1K-1.1K

Comparison of Algorithms: Total Weighted Scores

Algorithm	Stringer	Mean	95% CI
1-stage sampling, normalization table	Stringer I	7.03	(5.44, 8.21)
	Stringer II	7.56	(6.06, 8.54)
	Stringer III	7.99	(6.47, 8.93)
2-stage sampling, normalization table	Stringer I	7.02	(5.38, 8.21)
	Stringer II	7.56	(6.06, 8.56)
	Stringer III	7.99	(6.47, 8.93)
1-stage sampling, interval hull linear score	Stringer I	0.34	(0.12, 0.57)
	Stringer II	0.64	(0.43, 0.80)
	Stringer III	0.72	(0.51, 0.89)
2-stage sampling, interval hull linear score	Stringer I	0.34	(0.12, 0.56)
	Stringer II	0.64	(0.43, 0.80)
	Stringer III	0.72	(0.51, 0.89)
probability scoring	Stringer I	-0.47	(-0.73,-0.19)
	Stringer II	0.03	(-0.07, 0.13)
	Stringer III	0.44	(0.16, 0.69)

Comparison of Algorithms: Rank Acceptability Index

Algorithm	Stringer	Rank 1	Rank2	Rank3
1-stage sampling, normalization table	Stringer I	0.07	0.24	0.69
	Stringer II	0.25	0.54	0.21
	Stringer III	0.68	0.22	0.10
2-stage sampling, normalization table	Stringer I	0.08	0.24	0.68
	Stringer II	0.24	0.54	0.22
	Stringer III	0.68	0.22	0.10
1-stage sampling, interval hull linear score	Stringer I	0.01	0.04	0.95
	Stringer II	0.25	0.72	0.03
	Stringer III	0.74	0.24	0.02
2-stage sampling, interval hull linear score	Stringer I	0.01	0.04	0.95
	Stringer II	0.26	0.71	0.03
	Stringer III	0.74	0.25	0.01
probability scoring	Stringer I	0.00	0.00	1.00
	Stringer II	0.01	0.99	0.00
	Stringer III	0.99	0.01	0.00

BDAT

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Boeing Decision Analysis Tool (BDAT)

OVERVIEW:

Trade studies are needed to evaluate different designs and manufacturing processes in manufacturing industry. The decision factors for choosing a design or a manufacturing process can usually be categorized into numeric and subjective decision factors. Very often, experts provide subjective weight to each decision factor. Boeing Decision Analysis Tool (BDAT) provides several decision analysis algorithms to recommend an optimal decision by considering uncertainty of all various decision factors and weights. For more detailed explanation of decision factors, mapping table and weight that are input for decision analysis, please click the question marks below. Here is the BDAT Users Guide. For more technical details and additional questions, contact Shuguang Song at 206-304-8569.

DATA INPUT FILES

Decision Factors: Browse...

Add another Decision Factor

Mapping Table: Browse...

Weight file (optional): Browse...

Run Analysis

ANALYSIS RESULTS:

The analysis results will be displayed below. For a detailed explanation of these results, click the question mark above.

Analysis Results: Probability Score

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ANALYSIS RESULTS:

The analysis results will be displayed below. For a detailed explanation of these results, click the question mark above.

Result Set: probabilityScoreprobabilityScore

Export Analysis Results...

1_Mean_CI.csv

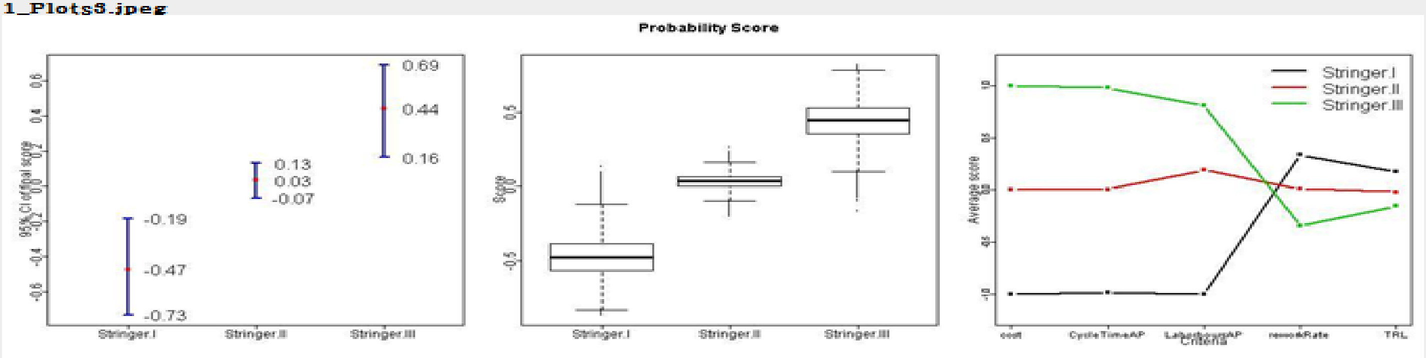
	Mean	95%CI-left	95%CI-right
Stringer.I	-0.47	-0.73	-0.19
Stringer.II	0.03	-0.07	0.13
Stringer.III	0.44	0.16	0.69

2_Rank_Acceptability_Index.csv

	Rank 1	Rank 2	Rank 3
Stringer.I	0	0	1
Stringer.II	0.01	0.99	0
Stringer.III	0.99	0.01	0

Show/Hide supporting data...

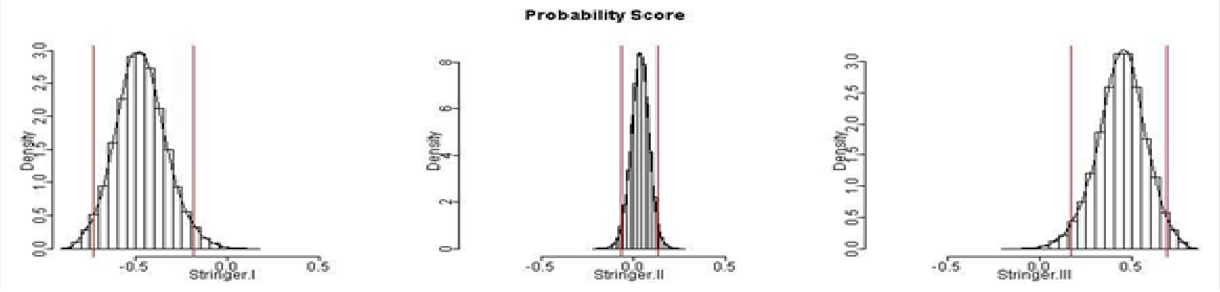
1_Plots5.jpeg



5_Value_Path.csv

	cost	CycleTimeAP	LaborhoursAP	reworkRate	TRL
Stringer.I	-1	-0.98	-1	0.34	0.18
Stringer.II	0	0	0.19	0.01	-0.02
Stringer.III	1	0.98	0.81	-0.35	-0.16

6_Histogram.jpeg



Analysis Results: 1-Stage Interval Hull

ANALYSIS RESULTS:

The analysis results will be displayed below. For a detailed explanation of these results, click the question mark above.

Result Set: 1stageintervalHull

Export Analysis Results...

1_Mean_CI.csv

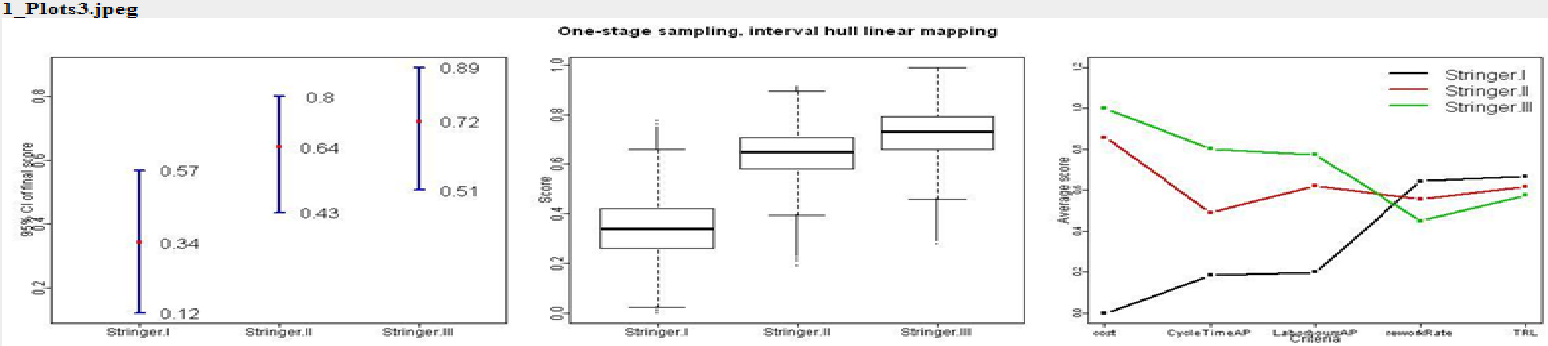
	Mean	95%CI-left	95%CI-right
Stringer.I	0.34	0.12	0.57
Stringer.II	0.64	0.43	0.8
Stringer.III	0.72	0.51	0.89

2_Rank_Acceptability_Index.csv

	Rank 1	Rank 2	Rank 3
Stringer.I	0.01	0.04	0.95
Stringer.II	0.25	0.72	0.03
Stringer.III	0.74	0.24	0.02

Show/Hide supporting data...

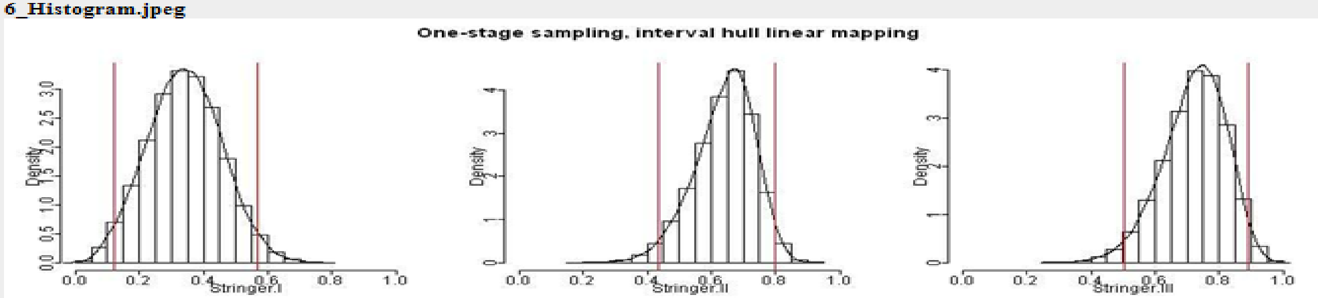
1_Plots3.jpeg



3_Value_Path.csv

	cost	CycleTimeAP	LaborhoursAP	reworkRate	TRL
Stringer.I	0	0.18	0.2	0.64	0.67
Stringer.II	0.86	0.49	0.62	0.56	0.62
Stringer.III	1	0.8	0.77	0.45	0.57

6_Histogram.jpeg



Analysis Results: 2-Stage Interval Hull

ANALYSIS RESULTS:
The analysis results will be displayed below. For a detailed explanation of these results, click the question mark above.

Result Set: 2stageintervalHull

Export Analysis Results...

1_Mean_CI.csv

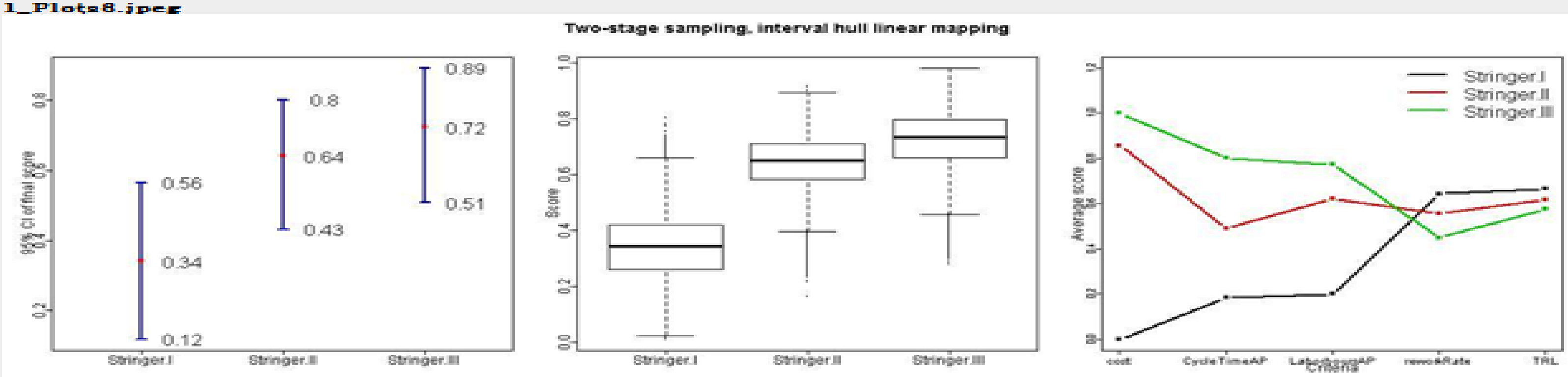
	Mean	95%CI-left	95%CI-right
Stringer.I	0.34	0.12	0.56
Stringer.II	0.64	0.43	0.8
Stringer.III	0.72	0.51	0.89

2_Rank_Acceptability_Index.csv

	Rank 1	Rank 2	Rank 3
Stringer.I	0.01	0.04	0.95
Stringer.II	0.26	0.71	0.03
Stringer.III	0.74	0.25	0.01

Show/Hide supporting data...

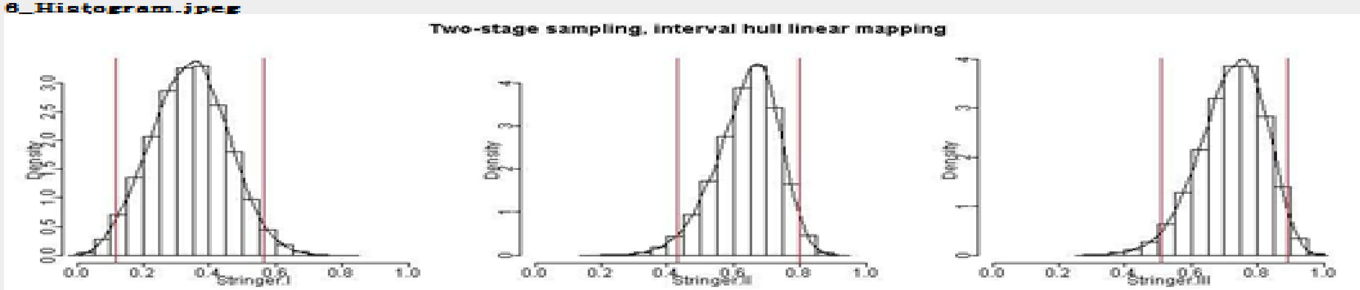
1_Plots6.jpeg



6_Value_Path.csv

	cost	CycleTimeAP	LaborhoursAP	reworkRate	TRL
Stringer.I	0	0.18	0.2	0.64	0.67
Stringer.II	0.86	0.49	0.62	0.56	0.62
Stringer.III	1	0.8	0.77	0.45	0.58

6_Histogram.jpeg



Conclusion and Discussion

- **Decision criteria: objective vs subjective**
- **Bayesian sampling approach**
- **Five MCDA algorithms:**
  - 1-stage sampling + normalization table
  - 2-stage sampling + normalization table
  - 1-stage sampling + interval hull linear score
  - 2-stage sampling + interval hull linear score
  - probability score (related to the pairwise winning index)
- **Extensive simulation is needed to compare the performance of the MCDA algorithms.**
- **It may be worth the development of some ensemble MCDA algorithm.**

Some References

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Questions?

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