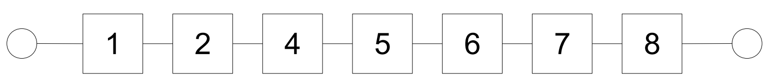
Chapter 5

5.1 Draw a reliability block diagram describing how to successfully perform an everyday task.

Consider the task of brushing your teeth. The following is a list of possible components for the block diagram:

1. get toothbrush
2. put toothpaste on toothbrush
3. put water on toothbrush
4. brush teeth
5. brush tongue
6. spit out toothpaste
7. rinse mouth
8. rinse toothbrush

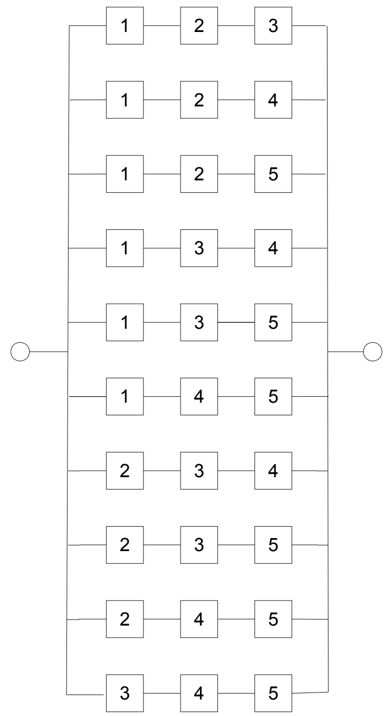
Below is the reliability block diagram.

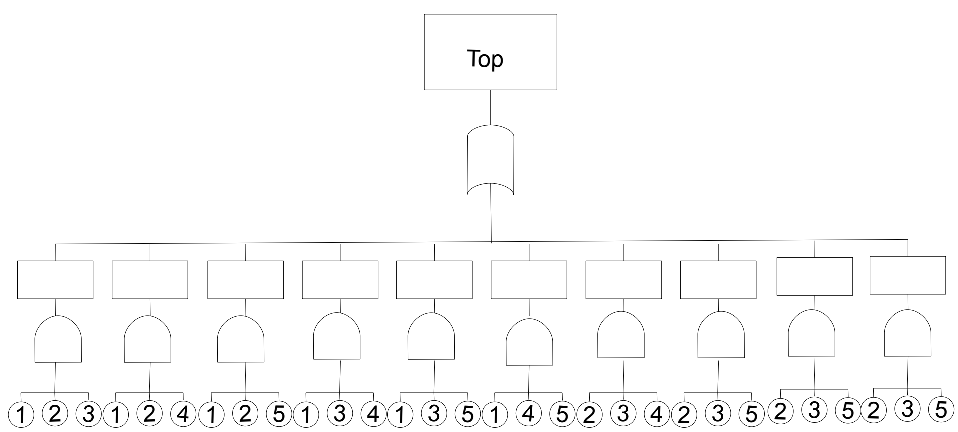


This a series system. Notice that component 3 is not essential for the cleaning of one's teeth, so it can be left out of the diagram.

For additional reading om the diagrams discussed in this chapter I recommend *System Reliability Theory* by Rausand and Høyland.

5.2 Draw the reliability block diagram and fault tree corresponding to a 3-of-5 system.





5.3 Determine the structure function for a 3-of-5 system.

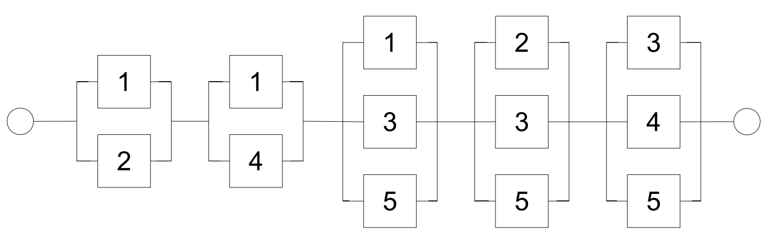
The structure for a k-of-n system is given by equation 5.1.

$$\begin{aligned}
\phi(\mathbf{x}) &= x_1x_2x_3(1-x_4)(1-x_5)+x_1x_2x_4(1-x_3)(1-x_5)\\
&\quad+x_1x_2x_5(1-x_3)(1-x_4)+x_1x_3x_4(1-x_2)(1-x_5)\\
&\quad+x_1x_3x_5(1-x_2)(1-x_4)+x_1x_4x_5(1-x_2)(1-x_3)\\
&\quad+x_2x_3x_4(1-x_1)(1-x_5)+x_2x_3x_5(1-x_1)(1-x_4)\\
&\quad+x_2x_4x_5(1-x_1)(1-x_3)+x_3x_4x_5(1-x_1)(1-x_2)\\
&\quad+x_1x_2x_3x_4(1-x_5)+x_1x_2x_3x_5(1-x_4)\\
&\quad+x_1x_2x_4x_5(1-x_3)+x_1x_3x_4x_5(1-x_2)\\
&\quad+x_2x_3x_4x_5(1-x_1)+x_1x_2x_3x_4x_5\\
&= x_1x_2x_3+x_1x_2x_4+x_1x_2x_5+x_1x_3x_4+x_1x_3x_5+x_1x_4x_5\\
&\quad+x_2x_3x_4+x_2x_3x_5+x_3x_4x_5-3x_1x_2x_3x_4-3x_1x_2x_3x_5\\
&\quad-3x_1x_2x_4x_5-3x_1x_3x_4x_5-3x_2x_3x_4x_5+6x_1x_2x_3x_4x_5
\end{aligned}$$

5.4 Draw the reliability block diagram corresponding to Fig. 5.9.

Using the 5 minimal cut sets we might draw the block diagram as

Block Diagram for Fig 5.9



5.5 Determine the minimal path sets and minimal cut sets for IE6 in Fig. 5.9. Calculate the structure function for IE6.

The minimal cut sets are {BE2, BE3, BE5}, {BE3, BE4, BE5}. The minimal path sets are {BE2, BE4}, {BE3}, {BE5}. To determine the structure for IE6 we can use either equations 5.3 or 5.4. Using equation 5.4 with the 3 minimal path sets we get

$$\begin{aligned}
\phi(\mathbf{x}) &= 1-(1-x_2x_4)(1-x_3)(1-x_5)\\
&= x_3+x_5+x_2x_4-x_3x_5-x_2x_3x_4-x_2x_4x_5+x_2x_3x_4x_5
\end{aligned}$$

5.6 Define the *structural importance* of component i in a coherent system of n components as

$$I_\phi(i) = \frac{1}{2^{n-1}} \sum_{\mathbf{x} \mid x_i = 1} [\phi(1_i, \mathbf{x}) - \phi(0_i, \mathbf{x})].$$

The sum is over the 2^{n-1} vectors for which $x_i = 1$. Calculate the structural importance of each component in Fig. 5.5.

For component 1

$(\cdot, x_2, x_3)$	$\phi(1, x_2, x_3) - \phi(0, x_2, x_3)$
$(\cdot 00)$	0
$(\cdot 01)$	1
$(\cdot 10)$	1
$(\cdot 11)$	1

$$I_\phi(1) = \frac{3}{2^{3-1}} = \frac{3}{4}$$

For component 2

$(x_1, \cdot, x_3)$	$\phi(x_1, 1, x_3) - \phi(x_1, 0, x_3)$
$(0 \cdot 0)$	0
$(0 \cdot 1)$	0
$(1 \cdot 0)$	1
$(1 \cdot 1)$	0

$$I_\phi(2) = \frac{1}{4}$$

For component 3

$(x_1, x_2, \cdot)$	$\phi(x_1, x_2, 1) - \phi(x_1, x_2, 0)$
$(00 \cdot)$	0
$(01 \cdot)$	0
$(10 \cdot)$	1
$(11 \cdot)$	0

$$I_\phi(3) = \frac{1}{4}$$

5.7 Derive Eq. 5.8 from Eq. 5.1 by assuming that each component has reliability $R_i(t) = R(t)$.

Beginning with equation (5.1),

$$P(\phi(x) = 1) = P(\sum_j (\prod_{i \in A_j} x_i)[\prod_{i \in A_j^c} (1 - x_j)] = 1)$$

We want to choose the subset A_j that is a minimum path set (i.e. $\phi(x) = 1$ for the elements in A_j). Therefore, we want at least k elements of A_j to be 1. Let s be the number of elements in A_j equal to 1. Therefore,

$$P((\prod_{i \in A_j} x_i)[\prod_{i \in A_j^c} (1 - x_j)] = 1) = P(s \geq k) = \sum_{s=k}^{n} \binom{n}{s} R(t)^s (1 - R(t))^{n-s} = \cdots$$

$$\cdots = 1 - \sum_{s=0}^{k-1} \binom{n}{s} R(t)^s (1 - R(t))^{n-s}$$

5.8 Calculate the hazard function for a series system with n components when each component lifetime has a Weibull distribution.

Let $C_i \sim Weibull(\lambda_i, \beta_i)$. By definition, the hazard function is $h_s(t) = \frac{f_s(t)}{R_s(t)}$. Using example 5.6 and $R_s = \prod_{i=1}^{n} R_i$, the hazard function is $h_s(t) = \sum_{i=1}^{n} \lambda_i \beta_i t_i^{\beta_i - 1}$

5.9 Show that the mean time to failure (MTTF) for a standby system with perfect switching is equal to the sum of the MTTFs for each component:

$$MTTF_S = \sum_{i=1}^{n} MTTF_i.$$

$MTTF_s = E[T_s] = E[T_1 + T_2 + \cdots + T_n] = E[T_1] + E[T_2] + \cdots + E[T_n] = \sum_{i=1}^{n} MTTF_i$

5.10 Suppose that each of the n components of a standby system with perfect switching has an $Exponential(\lambda)$ distribution. Show that the lifetime of the system has a $Gamma(n, \lambda)$ distribution.

$T_i \sim$ Exponential(λ) = Gamma($1, \lambda$). Let T_s denote the systems lifetime. Then $T_s = \sum_{i=1}^{n} T_i$. Therefore, since T_s is the sum of independent Gamma($1, \lambda$) random variables and using the result for gamma random variables in section B of the appendix, we have $T_s \sim$ Gamma(n, λ).

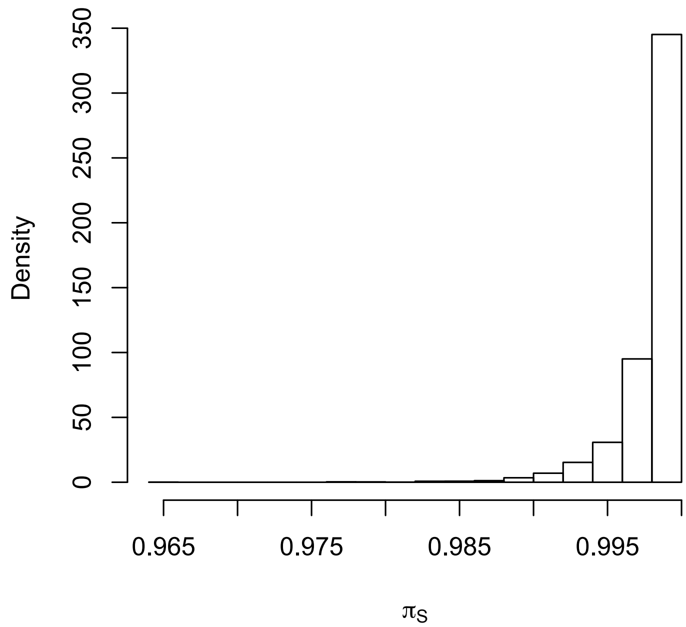
5.11 Reanalyze the data from Table 5.3 assuming that the prior distribution for the reliability of each component is $[\Gamma(1/3)]^{-1}(-\log(\pi_i))^{-\frac{2}{3}}$.

The posterior now becomes

$$p(\pi_1, \pi_2, \pi_3 \quad | \quad \mathbf{x}) \propto \pi_1^8(1-\pi_1)^2\pi_2^7(1-\pi_2)^2\pi_3^3(1-\pi_3)(\pi_1\pi_2\pi_3)^{10}(1-\pi_1\pi_2\pi_3)^2$$
$$[-\log(\pi_1)]^{-\frac{2}{3}}[-\log(\pi_2)]^{-\frac{2}{3}}[-\log(\pi_3)]^{-\frac{2}{3}}$$

			Quantiles				
Parameter	Mean	St.Dev	0.025	0.050	0.500	0.950	0.975
π_1	0.868	0.074	0.699	0.732	0.878	0.965	0.976
π_2	0.861	0.077	0.690	0.720	0.871	0.968	0.978
π_3	0.887	0.082	0.690	0.733	0.904	0.987	0.992
π_S	0.998	0.002	0.992	0.994	0.999	1.000	1.000

The following is a histogram of the posterior distribution on π_s:




```
mh <- function(theta, size, data){
    pi1 = theta[1]
    pi2 = theta[2]
    pi3 = theta[3]
    s = data[,2]
    f = data[,3]
    par = array(0, dim=c(size, 4))
    arate1 = 0; arate2 = 0; arate3 = 0
    post = function(theta){
        p1 = theta[1]; p2 = theta[2]; p3 = theta[3]
        ps = theta[1]*theta[2]*theta[3]
        val = p1^s[1] * (1-p1)^f[1] * p2^s[2] * (1-p2)^f[2] * p3^s[3] *
             (1-p3)^f[3] * ps^s[4] * (1-ps)^f[4] * (-log(p1))^(-2/3) *
             (-log(p2))^(-2/3) * (-log(p3))^(-2/3)
        return(val)
    }
    for(i in 1:size){
```

```
        pi1.star = runif(1)
        r = post(c(pi1.star, pi2, pi3)) / post(c(pi1, pi2, pi3))
        u = runif(1) <= r
        arate1 = arate1 + u
        pi1 = pi1.star*(u==1) + pi1*(u==0)
        pi2.star = runif(1)
        r = post(c(pi1, pi2.star, pi3)) / post(c(pi1, pi2, pi3))
        u = runif(1) <= r
        arate2 = arate2 + u
        pi2 = pi2.star*(u==1) + pi2*(u==0)
        pi3.star = runif(1)
        r = post(c(pi1, pi2, pi3.star)) / post(c(pi1, pi2, pi3))
        u = runif(1) <= r
        arate3 = arate3 + u
        pi3 = pi3.star*(u==1) + pi3*(u==0)
        pis = 1 - (1-pi1)*(1-pi2)*(1-pi3)
        par[i,] = c(pi1, pi2, pi3, pis)
    }
    arate = c(arate1, arate2, arate3); arate = arate/size
    list = list(par = par, accept = arate)
    return(list)
}

start <- data[,2]/data[,4]
sample <- mh(start[1:3], 10000, data)
plot(as.mcmc(sample$par))

#get rid of burn-in samples - calculate summary statistics
sample$par <- sample$par[-c(1:100),]
mu <- array(apply(sample$par, 2, mean), dim=c(4,1))
st.dev <- array(apply(sample$par, 2, sd), dim=c(4,1))
quant = rbind(quantile(sample$par[,1], c(.025, .05,.5,.95,.975)),
quantile(sample$par[,2], c(.025, .05,.5,.95,.975)),
quantile(sample$par[,3], c(.025, .05,.5,.95,.975)),
quantile(sample$par[,4], c(.025, .05,.5,.95,.975)))
summary = array(c(mu, st.dev, quant), dim=c(4,7))
colnames(summary) <- c("Mean", "Std Dev", "2.5%",  "5%", "50%",
"95%", "97.5%")
rownames(summary) <- c("pi1", "pi2", "pi3", "piS")
summary
```

```
hist(sample$par[,4], freq=F, xlab=expression(pi[S]), main="")
```

5.12 There are a variety of different measures of the reliability importance of a component (Rausand and Høyland, 2003). Birnbaum's measure of importance of the ith component at time t is

$$I_B(i \mid t) = \frac{\mathrm{d}R_S(t)}{\mathrm{d}\pi_i(t)}.$$

Birnbaum's measure is the partial derivative of the system reliability with respect to each component reliability $\pi_i(t)$. A larger value of $I_B(i \mid t)$ means that a small change in the reliability of the ith component results in a comparatively large change in the system reliability. Show that in a series system, Birnbaum's measure selects the component with the lowest reliability as the most important one.

The three Birnbaum's measures are: $I_{B1} = \pi_2\pi_3$, $I_{B2} = \pi_1\pi_3$, and $I_{B3} = \pi_1\pi_2$. Without loss of generality, suppose $\pi_1 < \pi_2 < \pi_3$. Based on the description of the measure in the exercise, we are looking for the largest value, which should correspond to π_1. Therefore, by comparing the different measures: $I_{B1} = \pi_2\pi_3 > \pi_1\pi_3 = I_{B2}$ if and only if $\pi_2 > \pi_1$. Which is true by our assumption. Also, $I_{B1} = \pi_2\pi_3 > \pi_1\pi_2 = I_{B3}$ if and only if $\pi_3 > \pi_1$. Which is again true by our assumption. Therefore, I_{B1} is the largest value and the procedure selected the most important component. This result still holds if $\pi_1 \leq \pi_2 < \pi_3$. It is trivial for the case that $\pi_1 = \pi_2 = \pi_3$.

5.13 Show how to calculate the posterior distribution for π_1, π_2, and π_3 using the data in Table 5.1 using simulation and the Metropolis-Hastings algorithm.

R code for a Metropolis-Hastings algorithm:

```
mh <- function(theta, size, data){
    pi1 = theta[1]
    pi2 = theta[2]
    pi3 = theta[3]
    s = data[,2]
    f = data[,3]
```

```
    par = array(0, dim=c(size, 3))
    arate1 = 0; arate2 = 0; arate3 = 0
    post = function(theta){
        p1 = theta[1]; p2 = theta[2]; p3 = theta[3]
        val = p1^s[1]*(1-p1)^f[1]*p2^s[2]*(1-p2)^f[2]*p3^s[3]*(1-p3)^f[3]
        return(val)
    }
    for(i in 1:size){
        pi1.star = runif(1)
        r = post(c(pi1.star, pi2, pi3)) / post(c(pi1, pi2, pi3))
        u = runif(1) <= r
        arate1 = arate1 + u
        pi1 = pi1.star*(u==1) + pi1*(u==0)
        pi2.star = runif(1)
        r = post(c(pi1, pi2.star, pi3)) / post(c(pi1, pi2, pi3))
        u = runif(1) <= r
        arate2 = arate2 + u
        pi2 = pi2.star*(u==1) + pi2*(u==0)
        pi3.star = runif(1)
        r = post(c(pi1, pi2, pi3.star)) / post(c(pi1, pi2, pi3))
        u = runif(1) <= r
        arate3 = arate3 + u
        pi3 = pi3.star*(u==1) + pi3*(u==0)
        par[i,] = c(pi1, pi2, pi3)
    }
    arate = c(arate1, arate2, arate3); arate = arate/size
    list = list(par = par, accept = arate)
    return(list)
}

start <- data[,2]/data[,4]
sample <- mh(start, 10000, data)
#get rid of burn-in and calculate summary statistics
plot(as.mcmc(sample$par))
sample$par <- sample$par[-c(1:50),]
mu <- array(apply(sample$par, 2, mean), dim=c(4,1))
st.dev <- array(apply(sample$par, 2, sd), dim=c(4,1))
quant = rbind(quantile(sample$par[,1], c(.025, .05,.5,.95,.975)),
    quantile(sample$par[,2], c(.025, .05,.5,.95,.975)),
    quantile(sample$par[,3], c(.025, .05,.5,.95,.975)),
```

```
    quantile(sample$par[,4], c(.025, .05,.5,.95,.975)))
summary = array(c(mu, st.dev, quant), dim=c(4,7))
colnames(summary) <- c("Mean", "Std Dev", "2.5%",  "5%", "50%", "95%", "97.5%")
rownames(summary) <- c("pi1", "pi2", "pi3", "piS")
summary
hist(sample$par[,4], freq=F, xlab=expression(pi[S]),
    main="Marginal Posterior Distribution from M-H")
```

The posterior distributions can be found in Table 5.2 and Fig. 5.15.

R code for a simulation:

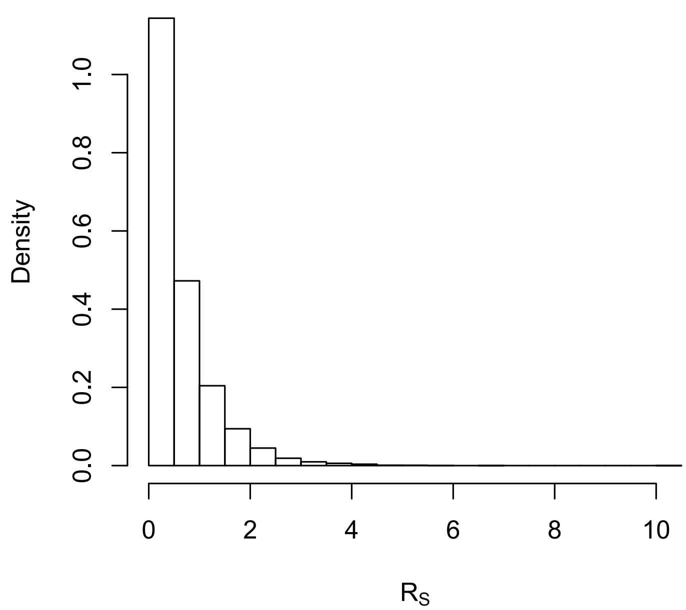
```
sim <- function(size){
    pi1 = rbeta(size, 9,3)
    pi2 = rbeta(size, 8,3)
    pi3 = rbeta(size, 4,2)
    pis = pi1*pi2*pi3
    pi = array(c(pi1,pi2,pi3, pis), dim=c(size,4))
    mu <- array(apply(pi, 2, mean), dim=c(4,1))
    st.dev <- array(apply(pi, 2, sd), dim=c(4,1))
    quant = rbind(quantile(pi[,1], c(.025, .05,.5,.95,.975)),
        quantile(pi[,2], c(.025, .05,.5,.95,.975)),
        quantile(pi[,3], c(.025, .05,.5,.95,.975)),
        quantile(pi[,4], c(.025, .05,.5,.95,.975)))
    summary = array(c(Mean=mu, Std.Dev=st.dev, quant), dim=c(4,7))
    colnames(summary) <- c("Mean", "Std Dev", "2.5%",  "5%", "50%",
        "95%", "97.5%")
    rownames(summary) <- c("pi1", "pi2", "pi3", "piS")
    list = list(pi = pi, summary = summary)
}
simulation <- sim(10000)
simulation$summary
hist(simulation$pi[,4], freq=F, xlab=expression(pi[S]),
    main="Marginal Distribution from Simulation")
```

			Quantiles				
Parameter	Mean	St.Dev	0.025	0.050	0.500	0.950	0.975
π_1	0.750	0.119	0.494	0.536	0.762	0.920	0.941
π_2	0.726	0.128	0.448	0.495	0.738	0.915	0.936
π_3	0.673	0.177	0.289	0.347	0.696	0.925	0.947
π_S	0.366	0.133	0.132	0.161	0.359	0.600	0.645

5.14 Assume a two-component series system. One component has an *Exponential*(3) prior distribution; the other has a *Weibull*(5, 2) prior distribution. Using simulation, determine the probability density function of the prior distribution for the system.

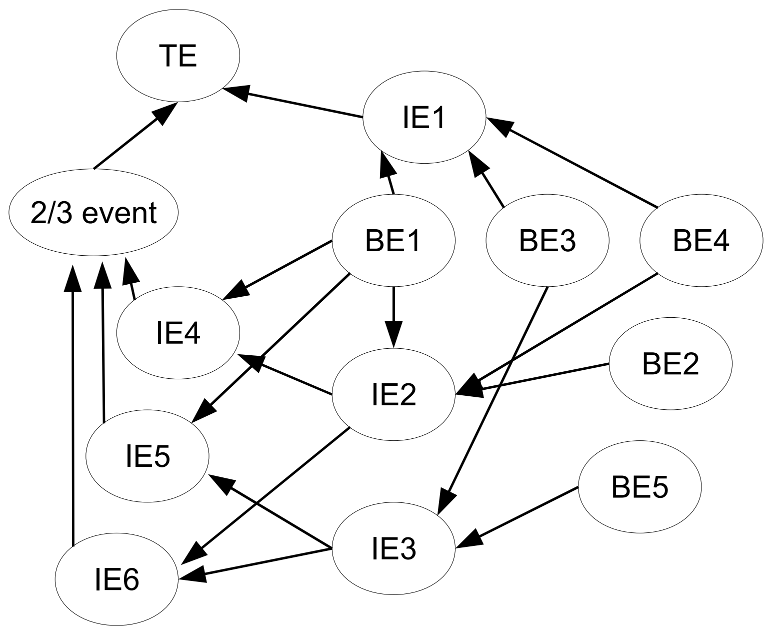
```
r1 = rexp(10000, 3)
r2 = rweibull(10000, 5,2)
rs = r1*r2
hist(rs, freq=F, xlab = expression(R[S]), main="")
mean(rs); sd(rs)
quantile(rs, c(.025, .05,.5,.95,.975))
```

			Quantiles				
Parameter	Mean	St.Dev	0.025	0.050	0.500	0.950	0.975
R_S	0.609	0.652	0.015	0.030	0.405	1.88	2.340



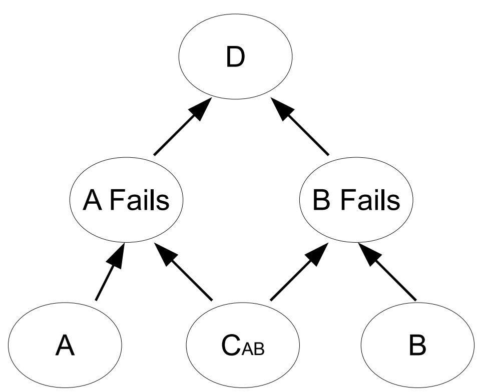
5.15 Translate the fault tree in Fig. 5.9 into a BN.

Bayesian Network for Fig. 5.9



5.16 Translate the fault tree in Fig. 5.24 into a BN. Write down the conditional probabilities specified by the fault tree.

Bayesian Network of Fig. 5.24



$$\mathbf{P}(B_F = 0 \,|\, C_{AB} = 0, B = 0) = 1 \qquad \mathbf{P}(A_F = 0 \,|\, C_{AB} = 0, A = 0) = 1$$
$$\mathbf{P}(B_F = 0 \,|\, C_{AB} = 1, B = 0) = 1 \qquad \mathbf{P}(A_F = 0 \,|\, C_{AB} = 1, A = 0) = 1$$
$$\mathbf{P}(B_F = 0 \,|\, C_{AB} = 0, B = 1) = 1 \qquad \mathbf{P}(A_F = 0 \,|\, C_{AB} = 0, A = 1) = 1$$
$$\mathbf{P}(B_F = 0 \,|\, C_{AB} = 1, B = 1) = 0 \qquad \mathbf{P}(A_F = 0 \,|\, C_{AB} = 1, A = 1) = 0$$

$$\mathbf{P}(D = 0 \,|\, A_F = 0, B_F = 0) = 1$$
$$\mathbf{P}(D = 0 \,|\, A_F = 1, B_F = 0) = 0$$
$$\mathbf{P}(D = 0 \,|\, A_F = 0, B_F = 1) = 0$$
$$\mathbf{P}(D = 0 \,|\, A_F = 1, B_F = 1) = 0$$

5.17 Suppose that the data in Table 5.3 come from a three-component parallel system. Using independent $Uniform(0,1)$ prior distributions for the reliability of each component, calculate the posterior distributions for the reliability of each component and the system.

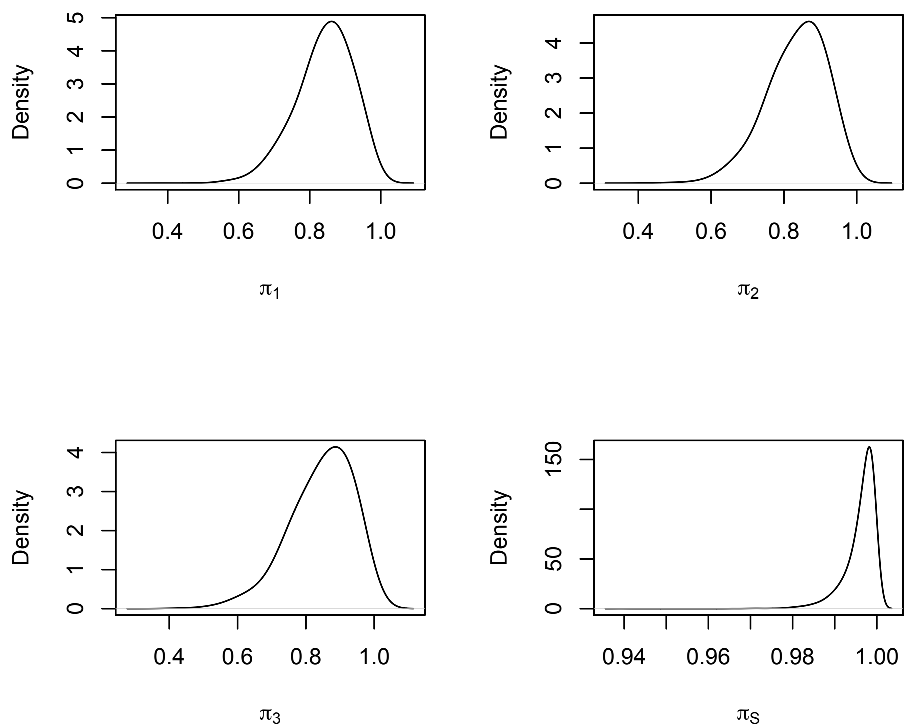
The formula for the reliability of the system in a parallel system is given on page 136. For the three component system in Table 5.3, we have

$$\pi_S = 1 - (1 - \pi_1)(1 - \pi_2)(1 - \pi_3)$$

The MCMC algorithm is similar to the one used in problems 11 and 13. Only the posterior function needs to be adjusted.

			Quantiles				
Parameter	Mean	St.Dev	0.025	0.050	0.500	0.950	0.975
π_1	0.841	0.078	0.669	0.696	0.850	0.953	0.962
π_2	0.834	0.081	0.655	0.684	0.845	0.948	0.959
π_3	0.845	0.092	0.624	0.676	0.858	0.968	0.978
π_S	0.996	0.004	0.986	0.989	0.997	1.000	1.000

Notice the difference between the posterior distribution for the reliability of this parallel system and the series system from the same data shown in Table 5.4.
The Kernel density estimates of the posterior distributions of the components and the system are shown below.



5.18 Suppose that we have a three-component system like that in Example 5.1, and suppose that each component has an $Exponential(\lambda)$ lifetime. Write an expression for the probability density function of the lifetime of the system.

$$f_i(t \mid \lambda) = \lambda e^{-\lambda t} \qquad F_i(t \mid \lambda) = 1 - e^{-\lambda t}$$

The reliability of the system can be derived combining equations 5.5

and 5.7.

$$\begin{aligned}
R_S(t) &= R_1(1-(1-R_2)(1-R_3)) \\
&= R_1R_2 + R_2R_3 - R_1R_2R_3 \\
&= 2R^2 - R^3 \qquad\qquad (\text{since } R_1 = R_2 = R_3) \\
1 - F_s(t) &= 2(1-F(t))^2 - (1-F(t))^3 \\
\frac{d}{dt}(1-F_s(t)) &= \frac{d}{dt}\left[1 - F(t) - F^2(t) + F^3(t)\right] \\
f_s(t) &= f(t) + 2f(t)F(t) - 3f(t)F^2(t) \\
&= \lambda e^{-\lambda t} + 2\lambda e^{-\lambda t}(1-e^{-\lambda t}) - 3\lambda e^{-\lambda t}(1-e^{-\lambda t})^2 \\
&= 4\lambda e^{-2\lambda t} - 3\lambda e^{-3\lambda t}
\end{aligned}$$

5.19 Reanalyze the BN in Fig. 5.22 with data from Tables 5.8 and 5.9 assuming that we have also observed 20 observations with $C_1 = 0, C_2 = 1, C_3 = 1$ that resulted in 6 system successes and 14 system failures.

We this information we can add $\pi_{FSS}^6(1-\pi_{FSS})^{14}$ to the likelihood and the posterior becomes

$$\begin{aligned}
p(\pi_1, \pi_2, \pi_3 \quad | \quad \mathbf{x}) \propto\ & \pi_1^8(1-\pi_1)^2\pi_2^7(1-\pi_2)^2\pi_3^3(1-\pi_3)\pi_{FSS}^6(1-\pi_{FSS})^{14}\pi_S^{10}(1-\pi_S)^2 \\
& [-\log(\pi_1)]^{-\frac{2}{3}}[-\log(\pi_2)]^{-\frac{2}{3}}[-\log(\pi_3)]^{-\frac{2}{3}} \\
& I[\pi_{FSS} \in (0.35, 0.85)]
\end{aligned}$$

Using the Metropolis-Hastings algorithm given below we obtain

			Quantiles				
Parameter	Mean	St.Dev	0.025	0.050	0.500	0.950	0.975
π_1	0.82	0.10	0.59	0.63	0.83	0.95	0.97
π_2	0.78	0.12	0.51	0.56	0.80	0.95	0.96
π_3	0.78	0.16	0.41	0.47	0.80	0.97	0.98
π_{FSS}	0.42	0.06	0.35	0.35	0.41	0.54	0.57
π_S	0.80	0.05	0.68	0.71	0.81	0.88	0.89

With this new information the 95% credible interval for π_{FSS} has narrowed from (.36, 0.84) in the example in the text to (0.35, 0.57).

```
mh <- function(theta, size, data){
    pi1 = theta[1]
```

```
pi2 = theta[2]
pi3 = theta[3]
pifss = theta[4]
s = data[,2]
f = data[,3]
par = array(0, dim=c(size, 5))
arate1 = 0; arate2 = 0; arate3 = 0; arate4 = 0
post = function(theta){
    p1 = theta[1]; p2 = theta[2]; p3 = theta[3]; pFSS = theta[4]
    ps = 0.95*p1*p2*p3 + 0.8*p1*p2*(1-p3) + 0.85*p1*(1-p2)*p3 +
        0.5*p1*(1-p2)*(1-p3) + pFSS*(1-p1)*p2*p3 + 0.4*(1-p1)*
        p2*(1-p3) + 0.55*(1-p1)*(1-p2)*p3 + 0.05*(1-p1)*(1-p2)*
        (1-p3)
    val = p1^s[1] * (1-p1)^f[1] * p2^s[2] * (1-p2)^f[2] * p3^s[3] *
        (1-p3)^f[3] * pFSS^6 * (1-pFSS)^14 * ps^s[4] *
        (1-ps)^f[4] * (-log(p1))^(-2/3) * (-log(p2))^(-2/3) *
        (-log(p3))^(-2/3)
    return(val)
}
for(i in 1:size){
    pi1.star = runif(1)
    r = post(c(pi1.star, pi2, pi3, pifss)) /
        post(c(pi1, pi2, pi3, pifss))
    u = runif(1) <= r
    arate1 = arate1 + u
    pi1 = pi1.star*(u==1) + pi1*(u==0)
    pi2.star = runif(1)
    r = post(c(pi1, pi2.star, pi3, pifss)) /
        post(c(pi1, pi2, pi3, pifss))
    u = runif(1) <= r
    arate2 = arate2 + u
    pi2 = pi2.star*(u==1) + pi2*(u==0)
    pi3.star = runif(1)
    r = post(c(pi1, pi2, pi3.star, pifss)) /
        post(c(pi1, pi2, pi3, pifss))
    u = runif(1) <= r
    arate3 = arate3 + u
    pi3 = pi3.star*(u==1) + pi3*(u==0)
    pifss.star = runif(1,.35,.85)
    r = post(c(pi1, pi2, pi3, pifss.star)) /
```

```
            post(c(pi1, pi2, pi3, pifss))
        u = runif(1) <= r
        arate4 = arate4 + u
        pifss = pifss.star*(u==1) + pifss*(u==0)
        pis = 0.95*pi1*pi2*pi3 + 0.8*pi1*pi2*(1-pi3) + 0.85*pi1*
            (1-pi2)*pi3 + 0.5*pi1*(1-pi2)*(1-pi3) + pifss*(1-pi1)*
            pi2*pi3 + 0.4*(1-pi1)*pi2*(1-pi3) + 0.55*(1-pi1)*(1-pi2)*
            pi3 + 0.05*(1-pi1)*(1-pi2)*(1-pi3)
        par[i,] = c(pi1, pi2, pi3, pifss, pis)
    }
    arate = array(c(arate1, arate2, arate3, arate4), dim=c(1,4))
    colnames(arate) = c("pi1", "pi2", "pi3", "piFSS")
    arate = arate/size
    return(list(par = par, accept = arate))
}
start <- c(data[1:3,2]/data[1:3,4], 6/14)
sample <- mh(start, 10000, data)
```

5.20 In Example 5.7, determine the probability that the item fails because of risk 1.

The probability that the item fails because of risk 1 is given by

$$\begin{aligned}
\mathbf{P}(T_1 < T_2) &= \int_0^\infty \mathbf{P}(T_2 > t \,|\, T_1 = t) f_{T_1} dt \\
&= \int_0^\infty e^{-\lambda_2 t} \lambda_1 e^{-\lambda_1 t} dt \\
&= \lambda_1 \int_0^\infty e^{-t(\lambda_1 + \lambda_2)} dt \\
&= \frac{\lambda_1}{\lambda_1 + \lambda_2}
\end{aligned}$$