Solutions to Selected Chapter 7 Exercises

7.2 We fit a logistic regression model in heating time and soaking time. We use the actual heating and soaking times (centered by their respective averages) as covariates in a full second order model, i.e., X_1, X_2, X_1X_2, X_1^2, and X_2^2, where X_1 and X_2 are the centered heating and soaking times, respectively. The regression coefficient of X_2 has a high posterior probability (0.992) of being different from zero, i.e., has an impact. The coefficients for X_2^2, X_1^2, and X_1X_2 are less important with posterior probabilities of 0.938, 0.920, and 0.908, respectively, of being different from zero. See the following reference for more details: B.P. Weaver and M.S. Hamada (2008), "A Bayesian Approach to the Analysis of Industrial Experiments: An Illustration with Binomial Count Data," *Quality Engineering*, 20, 269–280.

7.3 Using $K = 4 \approx 63^{0.4}$ equal probability bins, we find that 4.3% of the R^B test statistics exceed the 0.95 quantile of the $ChiSquared(3)$ reference distribution, which suggests no lack of fit.

7.14 a) We fit the model

$$Y_i \sim Poisson(\lambda_i t_i)\,,$$

where

$$\begin{aligned}\log(\lambda_i) \;=\;& \beta_1 + \beta_2 SYS2_i + \beta_3 SYS3_i + \beta_4 SYS4_i + \beta_5 SYS5_i \\ & + \beta_6 OTY2_i + \beta_7 OTY3_i + \beta_8 OTY4_i \\ & + \beta_9 VTY2_i + \beta_{10} VTY3_i + \beta_{11} VTY4_i + \beta_{12} VTY5_i + \beta_{13} VTY6_i \\ & + \beta_{14} SIZ2_i + \beta_{15} SIZ3_i + \beta_{16} OPM2_i\,.\end{aligned}$$

The covariates in the Poisson regression model above are dummy variables, e.g., $SYS2_i = 1$ if $SYS = 2$, etc. The summaries of the posterior distributions for the βs are given in the table below.

b) A 90% credible upper bound on the predicted number of failures in the next 10 years for a normally closed (OPM=1) 2- 10-inch (SIZ=2) air-driven (OTY=1) globe valve (VTY=5) in a power conversion system (SYS=3) is 99.

Parameter	Mean	Std Dev	Quantiles		
			0.025	0.500	0.975
β_1	-10.000	0.781	-11.740	-9.971	-8.618
β_2	0.9023	0.5226	-0.1079	0.8903	1.9800
β_3	1.030	0.490	0.121	1.012	2.065
β_4	1.214	0.551	0.161	1.204	2.343
β_5	0.317	0.572	-0.766	0.311	1.469
β_6	0.5966	0.5961	-0.6806	0.6254	1.6900
β_7	-1.213	0.248	-1.700	-1.211	-0.719
β_8	-2.560	0.497	-3.610	-2.529	-1.672
β_9	0.2025	0.7742	-1.3010	0.1882	1.7550
β_{10}	0.6059	0.7904	-0.9477	0.6105	2.1500
β_{11}	3.068	0.592	2.040	3.026	4.370
β_{12}	1.893	0.602	0.825	1.846	3.203
β_{13}	0.833	0.992	-1.246	0.875	2.677
β_{14}	-0.0039	0.2792	-0.5266	-0.0069	0.5741
β_{15}	1.625	0.316	1.021	1.618	2.262
β_{16}	-0.2065	0.1896	-0.5760	-0.2063	0.1617

7.17 For $X \sim Gamma(\alpha, \lambda)$, we can use $Y = \lambda X$ to remove λ as seen by $Y \sim Gamma(\alpha, 1)$. However, we cannot transform Y to remove α. Consequently, a Cox-Snell residual does not exist for the gamma distribution. The deviance residual for the ith observation y_i has the form:

$$\text{sign}(y_i - \mu_i)\sqrt{2[-\log(y_i - \mu_i) + (y_i - \mu_i)/\mu_i]}\,,$$

where $\mu_i = \alpha_i/\lambda_i^2$. See McCullagh and Nelder (1989) for more details.

7.24 Based on the predictive distribution summaries displayed in the table below, PCB type copper-tin-lead at 20°C is the recommended factor-level combination with the longest lifetime distribution.

PCB Type	Temp (° C)	Quantiles		
		0.025	0.500	0.975
copper-nickel-tin	20	251.2	700.2	1233.2
	60	55.0	157.9	278.5
	100	51.6	143.9	253.3
copper-nickel-gold	20	570.5	1636.9	2869.4
	60	89.8	533.9	929.1
	100	142.3	403.6	708.6
copper-tin-lead	20	587.4	1702.2	3031.9
	60	168.1	469.5	844.3
	100	130.5	372.8	646.8