Solutions to Selected Chapter 8 Exercises

8.1 We use the model $y_{ij} = -5 + \beta_i x_j + \epsilon$, where $\beta_i \sim Normal(\mu_\beta, \sigma_\beta^2)$ and $\epsilon \sim Norma(0, \sigma^2)$, for the jth observation of the ith LED. See the table below for posterior summaries of the model parameters. Using $K = 5$ equal probability bins, we find that 33.6% of the R^B test statistics exceed the 0.95 quantile of the $ChiSquared(4)$ reference distribution, which suggests some lack of fit. Using a threshold of -2.0 at 300 hours, a 90% credible interval for reliability is (0.3343, 0.7514) with a posterior median of 0.5505.

			Quantiles		
Parameter	Mean	Std Dev	0.025	0.500	0.975
β_1	0.004246	5.482E-4	0.003169	0.004245	0.005317
β_2	0.007738	5.528E-4	0.006640	0.007746	0.008833
β_3	0.007844	5.571E-4	0.006747	0.007838	0.008937
β_4	0.007461	5.543E-4	0.006385	0.007464	0.008572
β_5	0.009606	5.523E-4	0.008527	0.009608	0.010710
β_6	0.006429	5.500E-4	0.005347	0.006427	0.007494
β_7	0.007638	5.613E-4	0.006531	0.007635	0.008756
β_8	0.008719	5.525E-4	0.007629	0.008722	0.009819
β_9	0.011470	5.591E-4	0.010380	0.011470	0.012580
μ_β	0.007862	0.006230	-0.004482	0.007914	0.020440
σ	0.203600	0.024980	3.036E-4	0.200900	0.260200
σ_β	0.017710	0.005276	0.010830	0.016750	0.030620

8.4 Using $K = 9$ equal probability bins, we find that 32.5% of the R^B test statistics exceed the 0.95 quantile of the $ChiSquared(8)$ reference distribution, which shows lack of fit, as compared with the original model in Example 8.2, whose assessment is given below as the solution to Exercise 8.16. Consequently, we prefer the original model and do not use the alternative model to calculate $R(t)$ and $t_{0.1}$.

8.14 Using $K = 22$ equal probability bins, we find that 3.8% of the R^B test statistics exceed the 0.95 quantile of the $ChiSquared(21)$ reference distribution, which suggests no lack of fit.

8.16 Here, we consider only the fit of the model in Example 8.2. Using $K = 9$ equal probability bins, we find that 12.9% of the R^B test statistics exceed the 0.95 quantile of the $ChiSquared(8)$ reference distribution, which suggests no lack of fit.

8.17 It is doubtful that there is a closed form expression for the destructive measurement $z_i = D_i(t)$ with $D_i(t)$ defined in Eq. 8.21 when it is measured with a $Normal(0, \sigma^2)$ measurement error. Instead, we derive Eq. 8.23, the probability density function of z_i without measurement error as follows. We have $z = D(t) = \beta_0 - \beta_1(1/x)t$, where $x \sim Lognormal(\mu, \sigma^2)$, where the probability density function of x is

$$f(x \,|\, \mu, \sigma^2) \quad = \quad \frac{1}{x\sqrt{2\pi\sigma^2}} \exp\left[-\frac{1}{2\sigma^2}(\log(x) - \mu)^2\right].$$

We obtain the probability desntiy function of z by a change of variables. Because x can be expressed in terms of z by $x = \frac{\beta_1 t}{\beta_0 - z}$, we see that $\mathrm{d}x = \frac{\beta_1 t}{(\beta_0 - z)^2}\mathrm{d}z$. Consequently,

$$f(z \,|\, \mu, \sigma^2, \beta_0, \beta_1, t) \quad = \quad \frac{(\beta_1 t)}{(\beta_0 - z)^2} \frac{1}{\frac{\beta_1 t}{\beta_0 - z}\sqrt{2\pi\sigma^2}} \exp\left[-\frac{1}{2\sigma^2}\left(\log\left(\frac{\beta_1 t}{\beta_0 - z}\right) - \mu\right)^2\right].$$

which simplifies to Eq. 8.23.