Selected Solutions to Chapter 9 Exercises

9.1 Under a $Beta(1, 1)$ prior, the 0.90 posterior quantile of the 95% credible interval length for a sample size of 377 is 0.075. The 0.90 posterior quantile of the 95% credible interval length for a sample size of 3000 is 0.036, so that we use $n_{max} = 3000$. A bisection search finds that for a sample size of 2695, the 0.90 posterior quantile of the 95% credible interval length does not exceed 0.0375. That is $n_{optimal} = 2695$.

9.2 For (a) 5 out of 10 successes, $n_{optimal} = 1531$; (b) 50 out of 100 successes, $n_{optimal} = 1462$; (c) 9 out of 10 successes, $n_{optimal} = 1219$; and (d) 90 out of 100 successes, $n_{optimal} = 643$.

9.3 Under $X \sim Poisson(\lambda t)$, with a $Gamma(1, 1)$ prior for λ, $\alpha = 0.95$, $\gamma = 0.90$, $L_{target} = 0.05$, and $t_{max} = 100000$, we find using a bisection search that $t_{optimal} = 14339$. We can carry out the testing by allocating the total testing time of 14339 across multiple units that can be tested simultaneously, assuming that all the test units have a common λ.

9.4 Consider the data collection planning example, which focuses on reliability at time $t = 24$ months. Assuming that the lifetimes have a $LogNormal(\mu, \sigma^2)$ distribution, we use the following prior distributions: $\mu \sim Normal(4, 0.1)$ and $\sigma^2 \sim InverseGamma(20, 10)$. Letting $\gamma = 0.90$, $\alpha = 0.95$, and $L_{target} = 0.1$, we find that a sample size $n_{max} = 500$ meets the stated requirement, i.e., the probability of the $\alpha \times 100\%$ credible interval length of $R(24)$ not exceeding L_{target} is at least γ. A bisection search yields $n_{optimal} = 83$. The example in the chapter used prior distributions, which yielded a reliability prior distribution at 24 months with a median of 0.50 and a 0.95 probability interval of (0.00, 1.00). The prior distributions used in this exercise yielded a reliability prior distribution at 24 months with a median of 0.88 and a 0.95 probability interval of (0.79, 0.94).

9.7 We use the following less diffuse priors distributions:

$$\beta_0 \sim Normal(-7.3, (0.15/5)^2),$$
$$\beta_1 \sim Normal(7.5, (0.15/5)^2), \text{ and}$$
$$\sigma^2 \sim InverseGamma(100 \times 5, 0.11^2 \times 100 \times 5).$$

For $\gamma = 0.9$, $\alpha = 0.95$, $L_{target} = 0.1$, and reliability at time $t =$ 10,500 days, we check to make sure that the planning criterion for $n_i = 400, i = 1, 2, 3$ and $v_2 = 0.5$ meets the requirement. We use a GA that minimizes the total sample size, $n_1+n_2+n_3$, where instead of discarding (n_1, n_2, n_3) cases, which have a planning criterion ρ (i.e., the probability that the $\alpha \times 100\%$ credible interval length does not exceed L_{target}) that does not exceed γ, we penalize these cases by minimizing:

$$n_1 + n_2 + n_3 + [100(\gamma - \rho)/0.01]I(\rho < \gamma) + [25(\gamma - \rho)/0.01]I(\rho > \gamma).$$

A GA found the nearly optimal solution $v_{2,optimal} = 0.496$ and $\mathbf{n}_{optimal} = (n_1, n_2, n_3)$, where $n_1 = 145$, $n_2 = 50$, and $n_3 = 189$, whose penalized criterion is 384.