

Chapter 4

- 4.1 The $Beta(293, 0.5)$ prior for π yields a posterior with a median of 0.999 and a 95% credible interval of (0.990, 1.000), whose length is 0.010. The uniform prior (i.e., $Beta(1, 1)$) yields a posterior with a median of 0.997 and a 95% credible interval of (0.983, 1.000), whose length is 0.017.
- 4.3 The likelihood is $\prod_{i=1}^n (\lambda t_i)^{y_i} \exp(-\lambda t_i) / y_i! \propto \lambda^{\sum_{i=1}^n y_i} \exp(-\sum_{i=1}^n t_i \lambda)$ and the prior distribution is proportional to $\lambda^{\alpha-1} \exp(-\beta \lambda)$. Using Bayes' Theorem, the posterior distribution is proportional to $\lambda^{\alpha + \sum_{i=1}^n y_i - 1} \exp[-(\beta + \sum_{i=1}^n t_i) \lambda]$ so that $\lambda | \vec{y}, \vec{t} \sim Gamma(\alpha + \sum_{i=1}^n y_i, \beta + \sum_{i=1}^n t_i)$.
- 4.4 The $Gamma(5, 1)$ prior for λ yields a posterior with a mean of 2.85 and a 95% credible interval of (2.40, 3.05), whose length is 0.65. The $Uniform(0, 20)$ prior yields a posterior with a mean of 3.00 and a 95% credible interval of (2.52, 3.52), whose length is 1.00.
- 4.12 A 95% credible interval for μ is (2.65, 6.29) with a posterior median of 3.81. A 95% credible interval for λ is (1.01, 2.27) with a posterior median of 1.57. Using $K = 5$ equal probability bins, we find that 0.7% of the R^B test statistics exceed the 0.95 quantile of the $ChiSquared(4)$ reference distribution, which suggests no lack of fit. See the plot of the posterior median of reliability with 90% credible interval over the first 5 hours in Figure 1.
- 4.13 Using $K = 5$ equal probability bins, we find that 4.9% of the R^B test statistics exceed the 0.95 quantile of the $ChiSquared(4)$ reference distribution, which suggests no lack of fit. The DIC for the hierarchical model is 245.446 and the DIC for the constant failure probability model is 285.322. Based on DIC, we prefer the hierarchical model. We provide the WinBUGS code for the hierarchical in Table 1.

Table 1: WinBUGS code for exercise 4.13

```
#####
# Exercise 4.13
# EDG Hierarchical Model
# for success probabilities pi[]
# x[] number of failures in n[] trials

model
{
  for( i in 1 : N ) {
    z[i]<-ind[i]
    y[i]<-n[i]-x[i]
    y[i] ~ dbin(pi[i],n[i])
    pi[i] ~dbeta(delta,gamma)I(.0001,.9999)
  }

  #use InverseGamma
  delta<-1/rdelta
  gamma<-1/rgamma
  rdelta ~ dgamma(0.1,0.1)I(.001,1000)
  rgamma ~ dgamma(0.1,0.1)I(.001,1000)

}

Data

list(N = 63)

Inits

list(
  rdelta=.003,rgamma=.5,
  pi=c(
    .5,.5,.5,.5,.5,.5,.5,.5,.5,.5,
    .5,.5,.5,.5,.5,.5,.5,.5,.5,.5,
    .5,.5,.5,.5,.5,.5,.5,.5,.5,.5,
    .5,.5,.5,.5,.5,.5,.5,.5,.5,.5,
    .5,.5,.5,.5,.5,.5,.5,.5,.5,.5,
    .5,.5,.5,.5,.5,.5,.5,.5,.5,.5,
    .5,.5,.5)
)
#####
```

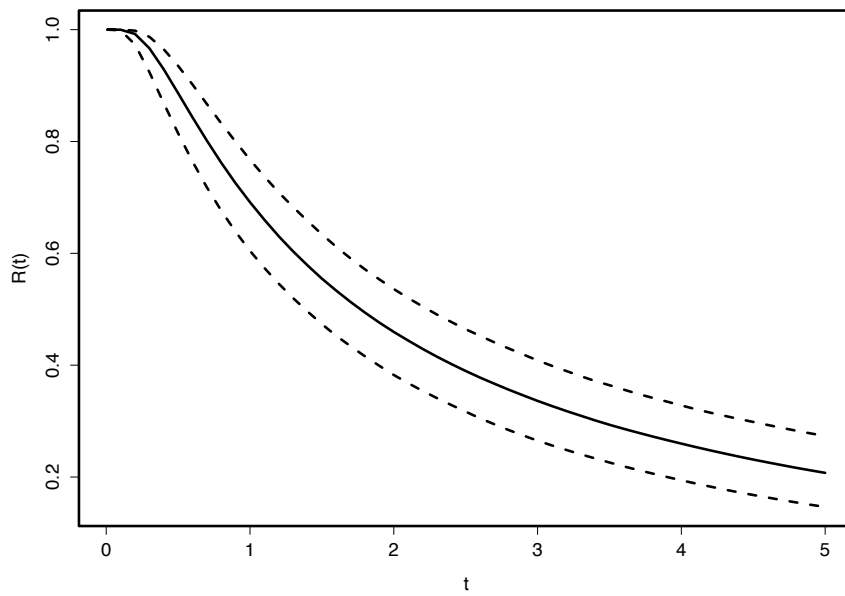


Figure 1: Exercise 4.12 median posterior of reliability and 95% credible intervals versus time t in hours.